# Comparison of Bidding Algorithms for Simultaneous Auctions

Seong Jae Lee

A Thesis Submitted to the
Brown University in Partial Fulfillment
of the Requirements for the Honors Degree
of Bachelor of Science

Department of Computer Science
Brown University

2007

Approved by
Advisory Committee

Chair

# Contents

## Abstract

Simultaneous auctions raise a challenge to bidders especially when there are substitutable or complementary goods. This thesis compares two classes of algorithms: marginal value-based algorithms and sampled average approximation-based algorithms, both of which are heuristics that optimize given a model of clearing prices. In the Trading Agent Competition, an annual event designed to promote research on trading agents, both algorithms are used for top players and showed almost even scores. Here, we show that sampled average approximation-based algorithms performs similar to or better than marginal value-based algorithms in both decision-theoretic setting and game-theoretic setting, especially when there is a high variance in the clearing price distribution. We claim this implies sampled average approximation-based algorithms are more adequate in a game-theoretic setting.

Thesis Supervisor: Amy Greenwald

Title: Assitant Professor of Computer Science

## Acknowledgements


First of all, I want to say thank professor Andries van Dam, who has been my advisor for 4 years. His comments helped me to adjust at Brown and become a passionate researcher. He was also a great teacher: his introductory computer science class was the first computer science class I took, which made me to study computer science, and the other class of his, the introductory computer graphics class, gave me a dream to pursue computer graphics.

I would also like to thank professor Amy Greenwald. I found this research topic from her artificial intelligence class two years ago. I still remember the day : building a TAC agent was the final project of her class, and our agent, atTACker, won the last place by a mistake. But Amy allowed me to work with her during the summer, and I have been working with her for two years since then. Without her, I could not have won at the Trading Agent Competition, published a paper, presented my work at a conference, or written this thesis. She has been a great advisor and mentor to me.

Thanks to Victor Naroditskiy, who has been working with me for two years. Fruitful discussions and encouraging support helped me to become a real researcher. Also, thanks to professor Meinolf Sellmann, who willingly agreed to be my thesis committee. Thanks to Korea Science and Engineering Foundation who gave me this great opportunity to study at Brown University. And last, but not least, thanks to my parents whom I love most.

# List of Tables

## List of Figures

# 1 Introduction

Simultaneous auctions raise challenges to bidders, especially when there are substitutable or complementary goods on sale. Susbstitutable goods are the ones with subadditive values. For example, when one is purchasing a travel package to Miami, making a hotel reservation on Hilton and one on Raddison on the same day would not increase the value of his package. In this case, a Hilton and a Radisson are substitutable goods. On the contrary, Complementary goods are the ones with superadditive values. For example, a round-trip flight ticket and a hotel reservation are complementary goods: a travel package with no flight tickets and a Radisson has no value.

The Trading Agent Competition, TAC, is an annual event to promote researches on bidding agents. In the TAC, eight players play against one other, and each player represents a travel agency. The goal of each player is to make profit by creating travel packages and selling them to assigned eight clients with different preferences. A travel package is composed of three kinds of goods: flight tickets, hotels, and entertainment ticekts, and each of the kind is purchased through different types of auctions. For example, hotel auctions close in a random order at each minute, and their clearing prices are decided to the sixteenth highest bidding prices. On the contrary, a flight ticket is sold through a continuous auction, and its clearing price is defined as a stochastic function of time.

In this thesis, I compare two classes of heuristics : sampled average approximation (SAA)-based algorithms and marginal value (MV)-based algorithms. Both of them showed a good performance in the TAC. Table 1 shows the successful agents in the TAC using SAA-based algorithms or MV-based algorithms.

| Agent | Performance | Algorithm |
|---|---|---|
| ATTac [SLSK01] | 2000 1st, 2003 1st | MV |
| Walverine [CLL+04] | 2003 final, 2004 2nd, 2005 3rd, 2006 2nd | MV |
| RoxyBot [GB01] | 2000 2nd, 2003 final, 2004 final | MV |
| RoxyBot [LGN07] | 2005 final, 2006 1st | SAA |

Table 1: Performance of SAA-based and MV-based algorithms in the TAC

This paper is orginized in four sections. In section 2, I explain SAA-based algorithms and MV-based algorithms. In section 3, I introduce three experiments: decision-theoretic, decision-theoretic with a noise, and game-theoretic. Each subsection contains experimental settings, results, and discussions. In section 4, we draw a final conclusion.

# 2 Bidding Algorithms

## 2.1 Sample Average Approximation

The sample average approximation (SAA) method solves stochastic optimization problems with the aid of Monte Carlo simulation [VAK+03]. The expected objective function of a problem is approximated by sampled estimates derived from a random sample. Here, we draw $S$ number of samples from a clearing price model and search the best bidding policy $b : X \to \mathbb{R}$ that maximizes:

$$\frac{1}{|S|}\sum_{s\in S} v(Y) - s(Y) \tag{1}$$

given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, and $Y := \{y \in X | s(y) < b(y)\}$.

However, there are infinite number of solutions for this equation. For example, if there is only one gods with possible clearing price 100, and if the utility of obtaining the goods is 1000, then bidding any number between 100 and 1000 is optimal. In this thesis, we developed two classes of algorithm, SAABottom and SAATop : SAABottom bids as low as possible and SAATop bids as high as possible. In case of SAATop, we set a constraint that determines the maximum limit of bidding price. The algorithms are described in the following section.

SAA-based algorithms are optimal when (i) the set of drawn samples is same as the clearing price model, and (ii) the clearing price model is the actual clearing price distribution. Unfortunately, it is impossible to fulfill these two conditions in most cases. The first condition does not hold if one does not count all the possible clearing prices, and it is even impossible if the model is continuous. Moreover, the second condition cannot be fulfilled in game-theoretic setting where the player does not know other players' strategies.

SAABottom suffers when the highest bid it considers submitting is below the clearing price. Similarly, SAATop suffers when the clearing price is higher than the highest price it expects. Because a SAA algorithm draws a finite number of samples from its distribution model, there is a possibility that it misses a clearing price within its samples. The possibility that all of the samples are smaller than the clearing price is the following :

$$\int_{-\bar{\infty}}^{\bar{\infty}} n(F(\bar{x}))^{n-1} f(\bar{x})(1 - G(x))d\bar{x} \tag{2}$$

while $f$ is the probability mass distribution of prediction, $F$ is the cumulative distribution function of prediction, and $G$ is the cumulative distribution function of the clearing prices.

In a single unit auction with perfect prediction, this probability is $1/(n+1)$.

$$\begin{aligned} &\int_{-\infty}^{\infty} n(F(x))^{n-1} f(x)(1 - F(x))dx \\ &= n\int_{-\infty}^{\infty} (F(x))^{n-1} f(x)dx - n\int_{-\infty}^{\infty} (F(x))^{n} f(x)dx \\ &= n\left[\frac{(F(x))^{n}}{n}\right]_{-\infty}^{\infty} - n\left[\frac{(F(x))^{n+1}}{n+1}\right]_{-\infty}^{\infty} \\ &= \frac{1}{n+1} \end{aligned} \tag{3}$$

Therefore, when SAABottom with $n$ scenarios bids its maximum, it may fail to complete its package with a high probability: the probability of losing a goods is $\frac{1}{n+1}$, and the probability of not losing a package composed of $d$ number of goods is $1 - (1 - \frac{1}{1+n})^{d}$. For example, when $n = 50$ and $d = 10$, it is 18.0%.

### 2.2 Marginal Value Bidding

Given a set of goods $X$, a valuation function $v : 2^X \to \mathbb{R}$, and a pricing function $s : 2^X \to \mathbb{R}$. The marginal value $\mu(x, s) = \mu(x, X, v, s)$ of good $x \in X$ is defined as follows:

$$\mu(x) = \max_{Y \subseteq X \setminus \{x\}} [v(Y \cup \{x\}) - s(Y)] - \max_{Y \subseteq X \setminus \{x\}} [v(Y) - s(Y)] \tag{4}$$

In other words, the marginal value of a good is the additional value derived from owning the good relative to the set of goods one can buy.

Greenwald showed that in a decision-theoretic setting, i) $\mu(x) > s(\{x\})$, when $x$ is within all optimal bidding sets, ii) $\mu(x) = s(\{x\})$, when $x$ is within some optimal bidding sets, and iii) $\mu(x) < s(\{x\})$, when $x$ is not in any of the optimal bidding sets [GN06]. Therefore, bidding marginal values on every goods in an optimal set is optimal assuming that there is only one clearing price. This is exactly what TargetMU algorithm does.

In this thesis, I used MV-based algorithms which performed well in the TAC and has been subject of many experiments [GN06] [OS06] [GB01]. TargetMU and TargetMU* are the algorithms of RoxyBot in the TAC 2000, BidEvaluator and BidEvaluator* are the algorithms of RoxyBot in the TAC 2002, and AverageMU and SampledMU are from the algorithm of ATTac. The algorithms are described in the following section.

### 2.3 Algorithm Descriptions

Here, I describe the algorithms used in this thesis. The number of samples from the clearing price model, or pricing functions, is determined so that each algorithm can play in the original TAC game: in the TAC, one should make a decision within a minute.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{SAABottom}} : X \to \mathbb{R}$ with a set of pricing functions $S$ is defined as :

$$b_{\mathsf{SAABottom}} = \text{argmax}_b \frac{1}{|S|} \sum_{s \in S} (v(Y) - s(Y)) - \epsilon \sum_{y \in Y} b(y) \tag{5}$$

while $Y := \{y \in X | s(y) < b(y)\}$. Here, $|S| = 50$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{SAATop}} : X \to \mathbb{R}$ with a set of pricing functions $S$ is defined as :

$$b_{\mathsf{SAATop}} = \text{argmax}_b \frac{1}{|S|} \sum_{s \in S} (v(Y) - s(Y)) + \epsilon \sum_{y \in Y} b(y),\ b(x) < c(x)\ \forall x \in X \quad (6)$$

while $Y := \{y \in X | s(y) < b(y)\}$. Here, $|S| = 50$ and $c(x) = \max(\max_S s(x), 350 + hb(x))\ \forall x \in X$, while $hb(x)$ is the maximum hotel bonus of that good across all its clients. $350 + hb(x)$ is the maximum profit one can get when $s(x) = 0$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{TargetMU}} : X \to \mathbb{R}$ with a set of pricing functions $S$ is defined as :

$$b_{\mathsf{TargetMU}}(x) = \mu(x, \frac{1}{S} \sum_{s \in S} s)\ \forall x \in A^* \quad (7)$$

while $A^* = \text{argmax}_{Y \subset X}(v(Y) - s(Y))$, and $Y := \{y \in X | s(y) < b(y)\}$. Here, $|S| = 50$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{TargetMU*}} : X \to \mathbb{R}$ with a set of pricing function $S$ is defined as :

$$b_{\mathsf{TargetMU*}}(x) = \mu(x, s^*)\ \forall x \in A^* \quad (8)$$

while $A^* = \text{argmax}_{Y \subset X}(v(Y) - s(Y))$, $s^*(x) = \frac{1}{S} \sum_{s \in S} s(x)\ \forall x \in A^*$, and $s^*(x) = \infty\ \forall x \notin A^*$. Here, $|S| = 50$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{BidEvaluator}} : X \to \mathbb{R}$ with two sets of pricing function $S$ and $S'$ is defined as :

$$b_{\mathsf{BidEvaluator}} = \text{argmax}_{b \in b_i} \frac{1}{|S'|} \sum_{s \in S'} (v(Y) - s(Y)) \quad (9)$$

while $Y := \{y \in X | s(y) < b(y)\}$, and $b_i = b_{\mathsf{TargetMU}}$ with a set of scenario $\{s_i\}, s_i \in S$. Here, $|S| = 25$ and $|S'| = 15$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{BidEvaluator*}} : X \to \mathbb{R}$ with two sets of pricing function $S$ and $S'$ is defined as :

$$b_{\mathsf{BidEvaluator*}} = \text{argmax}_{b \in b_i} \frac{1}{|S'|} \sum_{s \in S'} (v(Y) - s(Y)) \tag{10}$$

while $Y := \{y \in X | s(y) < b(y)\}$, and $b_i = b_{\mathsf{TargetMU*}}$ with a set of scenario $\{s_i\}, s_i \in S$. Here, $|S| = 25$ and $|S'| = 15$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{AverageMU}} : X \to \mathbb{R}$ with a set of pricing function $S$ is defined as :

$$b_{\mathsf{AverageMU}}(x) = \frac{1}{S} \sum_{s \in S} \mu(x, s) \ \forall x \in X \tag{11}$$

Here, $|S| = 15$.

Given a set of goods $X$, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{StraightMU}} : X \to \mathbb{R}$ with a set of pricing function $S$ is defined as :

$$b_{\mathsf{StraightMU}}(x) = \mu(x, \frac{1}{S} \sum_{s \in S} s) \ \forall x \in X \tag{12}$$

Here, $|S| = 50$.

# 3 Experiments

## 3.1 General Experimental Setting

The Trading Agent Competition is designed to promote research on trading agents. In a game, 8 players participate, and each player represents a travel agency with 8 clients. The goal of a player is to create travel packages that maximize its profit by procuring goods from different types of auctions. A travel package is composed of three kinds of goods - a round-trip flight, a hotel reservation, and entertainment tickets -, which are purchased from different kinds of auctions. The profit of a player is determined by the preferences of its clients, travel packages, and the cost of procured goods. Precisely, the profit per a client is determined as :

$$\text{profit} = 1000 - \text{travel penalty} - \text{cost} + \text{hotel bonus} + \text{fun bonus} \tag{13}$$

while travel penalty is defined as 100 times the sum of i) the difference of preferred arrival date and actual arrival date, and ii) the difference of preferred departure date and actual departure date, hotel bonus is the bonus when a client gets the nicer hotel, and fun bonus is the bonus when a client gets entertainment tickets. Hotel bonus and fun bonus is determined randomly for each client.

A travel package should be within day 1 and day 5, thus there are 4 inbound flight tickets and 4 outbound flight tickets. There are two kinds of hotel, Tampa Towers and Shoreline Shanties, and every client prefer staying at the former. Eight auctions are held for each hotel and each day. Finally, there are three kinds of entertainment tickets for each day.

Flights can be purchased continuously, and the clearing prices is defined as a stochastic function of time with a hidden parameter. Entertainment tickets can be purchased in continuous double auctions from other players - at the beginning of a game, entertainment tickets are distributed to players randomly. Finally, hotel auctions close at each minute in a random order, and each auction distributes 16 rooms of same type of hotel on same day with the 16th highest bidding price as a clearing price. The 16th highest bidding price of each auction is notified to players every minute.

We modified this game setting into a simultaneous one: all auction closes simultaneously. To make it simpler, flight ticket prices are fixed to 325 and entertainment tickets are removed.

### 3.2 Experiment with Perfect Prediction

#### 3.2.1 Experimental Setting

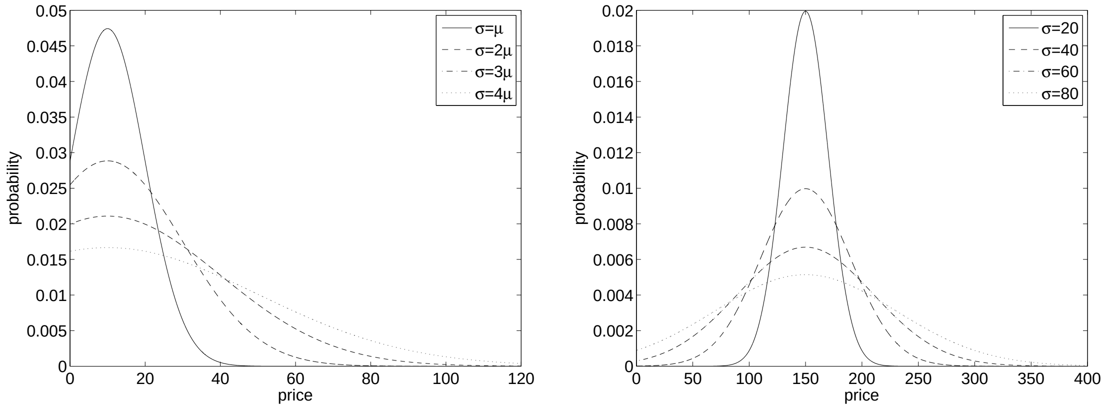
In the first experiment, prediction and clearing prices are sampled from the same normal distribution with mean $\bar{\mu} = (10, 50, 50, 10, 40, 110, 110, 40)$ and standard deviation $\bar{\sigma} = \lambda\bar{\mu}$. $\bar{\mu}$ is close to the TAC's competitive equilibrium price. Each auction's clearing price is sampled independently. When the price is sampled below zero, it is resampled. Figure 1(a) shows the probability distribution of the clearing prices of a hotel with $\mu = 10$. I omitted other distribution graphs, because the shape of distributions would be basicaly same to the shown graph, while the unit of x axis would be proportional to their $\mu$. I ran four experiments with parameter $\lambda = \{1, 2, 3, 4\}$, 1000 games for each.

In the second experiment, prediction and clearing prices are sampled from the normal distribution with mean $\bar{\mu} = (150, 150, 150, 150, 250, 250, 250, 250)$ and standard deviation $\sigma$. Again, each auction's clearing price is sampled independently. When the sampled price is below zero, it is resampled. Figure 1(b) shows the probability distribution of the clearing prices of a hotel with $\mu = 150$. With low variance, the shape of the distribution is same for other auctions. When variance is high enough, the probability distribution would be different due to resampling. I ran eight experiments with parameter $\sigma = \{1, 20, 40, 60, 80, 100, 120, 140\}$, 1000 games for each.

#### 3.2.2 Result

The score mean of each algorithm is shown in Figure 2. Confidence intervals of score mean and detailed game result tables are shown at the Appendix A.

In the first experiment, SAATop, BidEvaluator*, and TargetBidder* perform best when $\sigma = 1$. As variance increases, the scores of BidEvaluator* and TargetBidder* drop faster than those of SAATop and SAABottom, making the latter top players in a high variance setting. One should note that the drastic fall of scores is caused not only by the changes on variance but also by the changes on mean: since the sampling method truncates the samples with




(a) $\bar{\mu} = (10, 50, 50, 10, 40, 110, 110, 40)$ (b) $\bar{\mu} = (150, 150, 150, 150, 250, 250, 250, 250)$

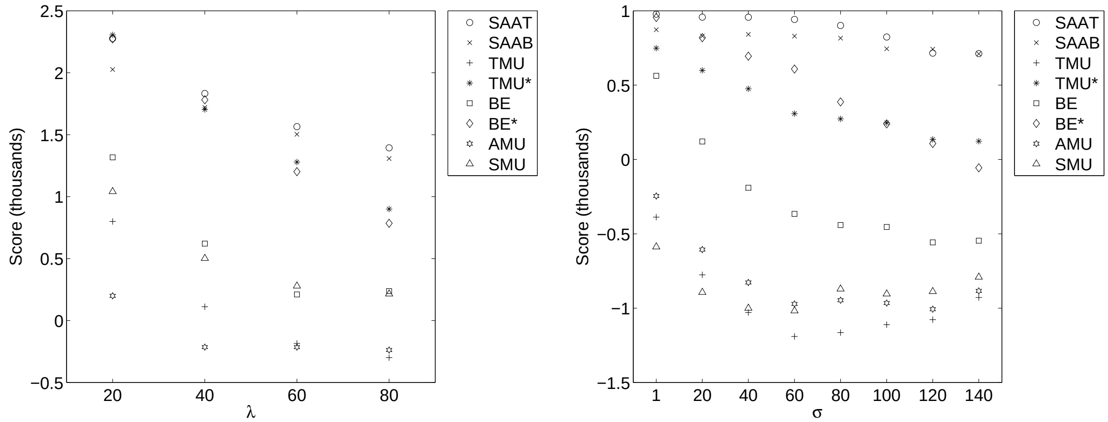
Figure 1: Clearing price distribution, experiment with perfect prediction

negative price, the mean of the model increases as its variance does. For example, when $\lambda = 4$, the actual mean of the distribution is (37, 180, 180, 39, 145, 384, 378, 145).

In the second experiment, SAATop and BidEvaluator* perform best when $\sigma = 1$. However, the scores of SAA-based algorithms drop slower than those of MV-based algorithms as variance increases. When $\sigma = 140$, the order of the algorithms is SAATop, SAABottom, TargetBidder*, BidEvaluator*, BidEvaluator, TargetBidder, AverageMU and StraightMU, starting from the highest score. This is consistent with the order of algorithms in the first experiment, $\lambda = 80$.

### 3.2.3 Discussion

There is a score gap between SAATop and SAABottom. With the same scenario set, they aim the same set of goods for each scenario. The only difference between them is their bidding prices: SAATop places as high as possible to win the same goods, while SAABottom places as low as possible to win those. Therefore, SAABottom is more likely to lose goods that it was sure that it would win them. When one is bidding on a set of complementary goods, losing a goods means not only losing the expected profit, but also losing all the complementary goods it purchased: in this experiment, SAABottom has to throw a pair of round trip flight tickets out.




(a) $\bar{\mu} = (10, 50, 50, 10, 40, 110, 110, 40)$ (b) $\bar{\mu} = (150, 150, 150, 150, 250, 250, 250, 250)$

Figure 2: Score mean, experiment with perfect prediction

We can calculate the number of unexpected failure of winning aimed goods for SAABottom from Equation 2. In the second experiment, SAABottom would unexpectedly fail to complete a travel package with probability $\frac{1}{51}^{d}$. Since the number of used hotels is almost a half of the number of used flights, we can assume $d = 1$ (Table 7). Also, Figure 3 shows that SAA-based algorithms bid more than 95% of goods with the maximum price when $\sigma = 20$: the right part of bidding price distribution resembles the highest sampled price distribution. Therefore, the number of unexpected failure per a travel package would be about $\frac{1}{51}$. Considering that the average number of completed travel packages for SAATop is 7.1, the number of unexpected failure per a game would be about 0.14 ($7.1 \times \frac{1}{51}$). The number of null package for SAABottom is greater than that of SAATop by 0.22. We can say a great part of the mal-performance of SAABottom came from this unexpected failure.

For the second experiment with $\sigma = 20$ (Table 7), the number of unused flight tickets per game is 0.01 for SAATop, and 0.38 for SAABottom (Table 7). SAABottom buys two flight tickets (650), while SAATop buys two flight tickets (650) and a hotel reservation (200) but compensated with utility (6997/(8-0.9)=985). Therefore, each failure on aimed hotel makes SAABottom earns 785 less than SAATop. Since the difference on the number of unused flight tickets is 0.37, we can infer SAABottom lost in hotel auctions 0.19 more per game. This gives almost 150 points off from SAABottom. From the table, we can see the score difference between two algorithms is 193.

However, in the second experiment, the performances of SAATop and SAABottom become similar as variance increases. This happens because the bidding price of SAATop and SAABottom becomes similar. Figure 3 shows that when $\sigma = 80$, the bidding price distribution of SAABottom and that of SAATop shows more similarity. Table 13 shows that the difference of the number of unused flight between SAATop and SAABottom is 0.05 with $\sigma = 140$, while it is 0.22 with $\sigma = 20$.

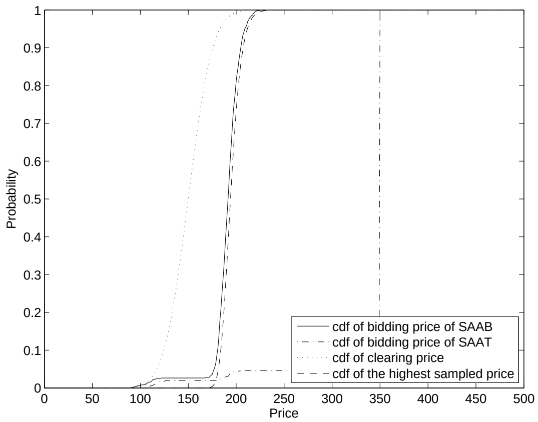
Another thing to note is that Figure 3 shows that the maximum amount SAATop bids is more than the marginal value of the good. Since the maximum possible marginal value is 350 plus a hotel bonus (utility 1000 minus two flight tickets 650 plus a hotel bonus), MV-based algorithms will never bid more than 350. On the contrary, the maximum amount that an optimal algorithm can bid is 1000.

Both experiments show that SAA-based algorithms are more tolerant to the variance of the clearing price distribution compared to MV-based algorithms. The performance of the latter decrease faster than that of the former as variance increases. Figure 3(c) and Figure 3(d) shows two best MV-based algorithms are more likely to lose goods in the higher variance setting.

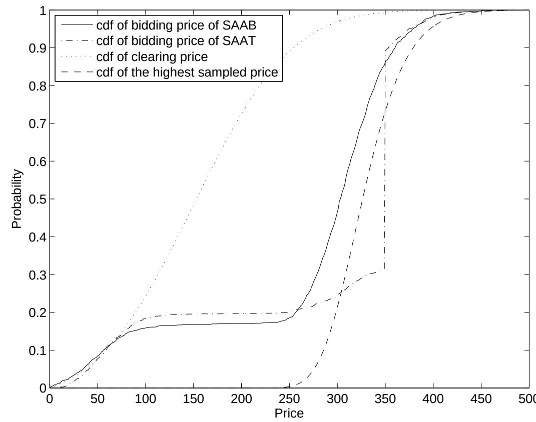
## 3.3 Experiment with Noisy Prediction

### 3.3.1 Experimental Setting

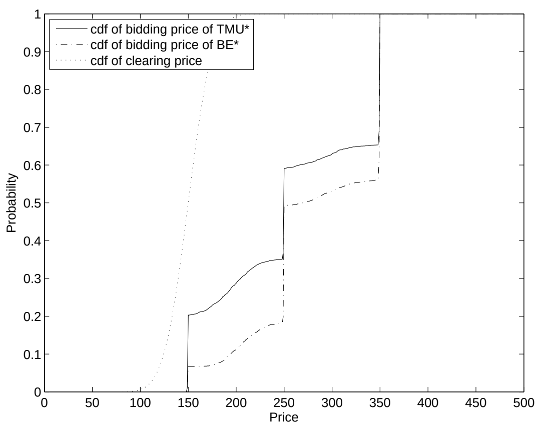
In this experiment, prediction is sampled from the normal distribution with mean $\bar{\mu}$ = (150, 150, 150, 150, 250, 250, 250, 250) and the given parameter $\sigma$. The clearing price is shifted by the given parameter $\lambda$ from the same distribution. For example, when $\sigma = 20, \lambda = -40$, prediction is sampled from the normal distribution with mean $\bar{\mu}$ = (150, 150, 150, 150, 250, 250, 250, 250) and $\sigma = 20$, and clearing price is sampled from the normal distribution with mean $\bar{\mu}$ = (110, 110, 110, 110, 210, 210, 210, 210) and $\sigma = 20$. I ran experiments with parameter $\lambda$ = {-40, -30, -20, -10, 10, 20, 30, 40} and $\sigma$ = {20, 80}, 500 games for each.



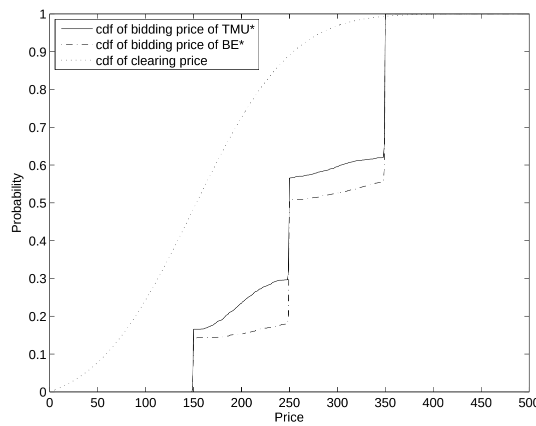

(a) SAATop and SAABottom, $\sigma = 20$




(b) SAATop and SAABottom, $\sigma = 80$




(c) TargetBidder* and BidEvaluator*, $\sigma = 20$




(d) TargetBidder* and BidEvaluator*, $\sigma = 80$

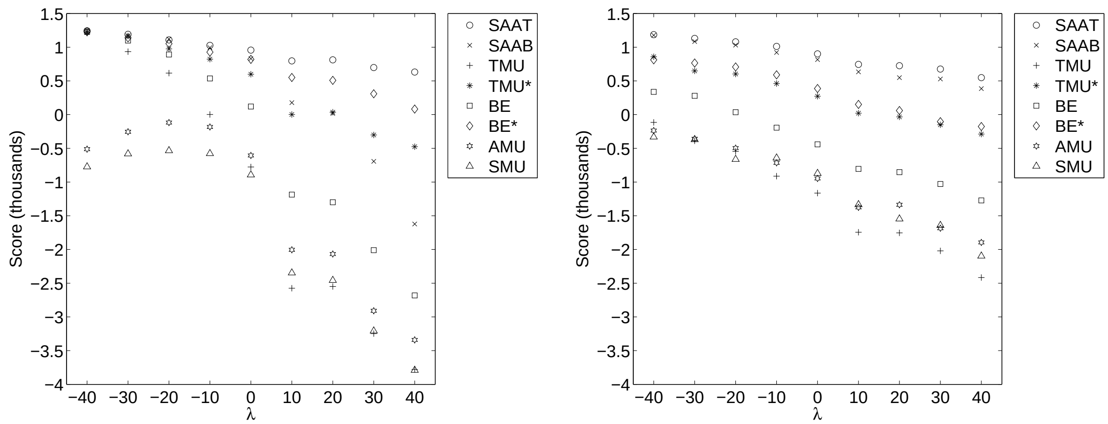
Figure 3: Cdf of bidding prices for cheap hotels, experiment with perfect prediction

#### 3.3.2 Result

The mean score is shown in Figure 4. Confidence intervals of score mean and detailed game result tables are shown at the Appendix B.

In the first experiment with $\sigma = 20$ (Figure 4(a)), all the algorithms except AverageMU and StraightMU perform best when $\lambda = -40$, that is, when the mean of the clearing price is lower than the mean of prediction by 40. As $\lambda$ increases, the order of the algorithms changes to SAATop, BidEvaluator*, TargetBidder*, SAABottom, BidEvaluator, AverageMU, StraightMU and TargetBidder from the highest score. In general, the score of an algorithm is higher when $\lambda$ is negative, although there are exceptions for AverageMU and StraightMU.

In the second experiment with $\sigma = 80$ (Figure 4(b)), the order of the algorithms are consistent across all $\lambda$ in the order of SAATop, SAABottom, BidEvaluator*, TargetBidder*, BidEvaluator, AverageMU, StraightMU and TargetBidder from the highest score. In general, the score of an algorithm is higher when $\lambda$ is negative.




(a) $\sigma = 20$ (b) $\sigma = 80$

Figure 4: Score mean, experiment with noisy prediction

### 3.3.3 Discussion

In the first experiment, there is very small difference in score between all the algorithms except AverageMU and StraightMU when $\lambda = -40$. Table 14 shows that these algorithms created a single hotel package in the most of the time, because the number of used flights is twice the number of used hotels. Moreover, they bid only the hotels they wanted to complete, because the number of purchased hotels is same as the number of used hotels. With these bidding polices, the problem can be reduced to the bidding problem with a single clearing price. When $\sigma = 20$ and $\lambda = -40$, the clearing price will be lower than the mean price of scenarios most of the time, because 95% of the clearing prices would be below $\mu - 20$, and 95% of the prices in a scenario would be above. Since TargetBidder bids at least the mean price of the scenarios on aimed goods, it will hardly fail winning its bids. In a similar way, BidEvaluator will also hardly fail winning its aimed bids, since

BidEvaluator bids at least the price of a scenario. Because TargetBidder* and BidEvaluator* bids higher than TargetBidder and BidEvaluator respectively, these four algorithms - TargetBidder, TargetBidder*, BidEvaluator and BidEvaluator* - will be sub-optimal.

The drastic fall of SAABottom can also be explained in the same way. The bidding price of SAABottom is less than the maximum price of its prediction plus $\epsilon$, because SAABottom assumes that it would win any of the bids with this price. When $\lambda = 40$ and $\sigma = 20$, the clearing price will be greater than most of the price SAABottom bids, since 95% of the clearing prices would be above $\mu + 20$, while 95% of the prices in a scenario would be below. If we assume most of the packages are single hotel packages, the probability that SAABottom unexpectedly fails to complete a travel package is 41.31% from Equation 2. Table 21 shows the number of hotels won by SAABottom is only 54.93% of that by SAATop. On the contrary, when $\lambda = 40$ and $\sigma = 80$, the ratio is 5.50%. Table 29 shows the number of hotels won by SAABottom is 94.12% of that by SAATop.

The scores of AverageMU and StraightMU increase for a while and then decrease as $\lambda$ increases. increase is caused by winning their bids too much, and the decrease is due to losing their bids too much.

## 3.4 Experiment with CE Prediction

### 3.4.1 Experimental Setting

Here, I present the experiments with game-theoretic settings. More than one players place bids on each auction, and those who placed bids greater than or equal to the sixteenth highest bidding price will win the goods with the price. Because we cannot know what the clearing prices will be in a game-theoretic setting, the quality of prediction matters. There is a prediction method to calculate the competitive equilibrium (CE) price of the TAC game proposed by TAC agent Walverine [CLL+04]. Here, I used its modified algorithm, using a simultaneous ascending auction technique, which is used in RoxyBot-06 [LGN07]. This algorithm increases a price while the total demand is greater than the total supply starting from zero price (Equation 14).

$$P_{n+1} = P_n + \text{MAX}(0, \alpha P_n(\text{demand} - \text{supply})), P_0 = 0 \tag{14}$$

It is different from the prediction used in previous experiments: it is not a normal distribution, and the price of an auction is dependent on those of other auctions. For example, the prices of hotels on the same day have a high covariance, because they are substitutable goods.

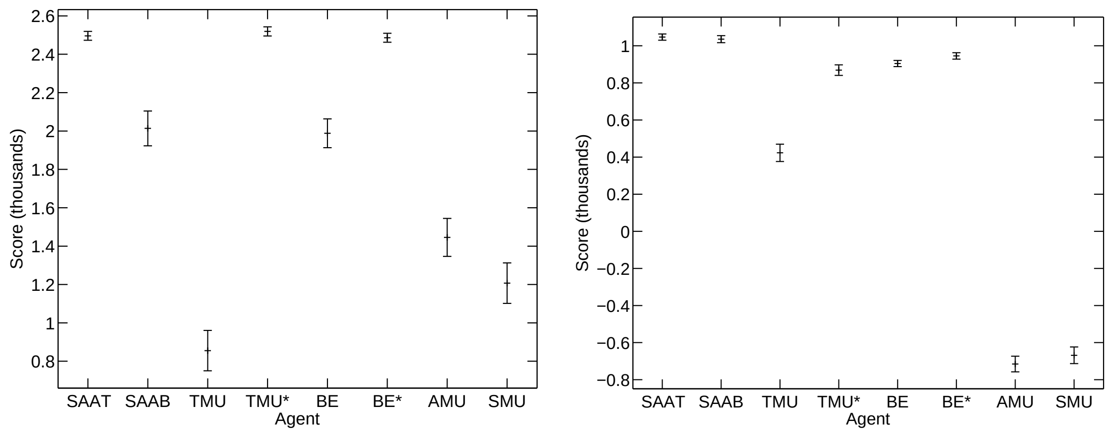
For the first experiment, the number of palyers in the game is eight as a normal TAC game does. For the second experiment, the number of players in the game follows a binomial distribution with parameter 32 and 0.5. In the second experiment, a player chooses its algorithm randomly. 1500 games were played for the first experiment, and 1046 games were played for the second experiment.

### 3.4.2 Result

The mean score is shown in Figure 5. Detailed game result tables are shown at the Appendix C. In the first experiment, SAATop, TargetBidder* and BidEvaluator* are the top algorithms, follwed by SAABottom and BidEvaluator. In the second experiment, SAATop and SAABottom are the top algorithms, followed by BidEvaluator*, BidEvaluator, and TargetBidder*.

Figure 6(a) and Figure 6(c) show the distribution of CE prediction and that of clearing prices for the first experiment. The mean of prediction was (2, 57, 57, 2, 30, 97, 97, 30), and its standard deviation was (7, 23, 23, 7, 24, 14, 14, 25). The mean of clearing prices was (1, 53, 53, 1, 29, 95, 96, 31), and its standard deviation was (4, 29, 29, 4, 29, 20, 20, 30).

Figure 6(b) and Figure 6(d) show the distribution of CE prediction and that of clearing prices for the second experiment. The mean of CE prediction was (137, 155, 155, 137, 239, 255, 255, 239), and its standard deviation was (55, 45, 45, 55, 60, 53, 53, 60). The mean of clearing price was (123, 127, 126, 127, 229, 219, 219, 230), and its standard deviation was (59, 52, 52, 56, 73, 71, 72, 70).




(a) 8 players

(b) B(32, 25) players

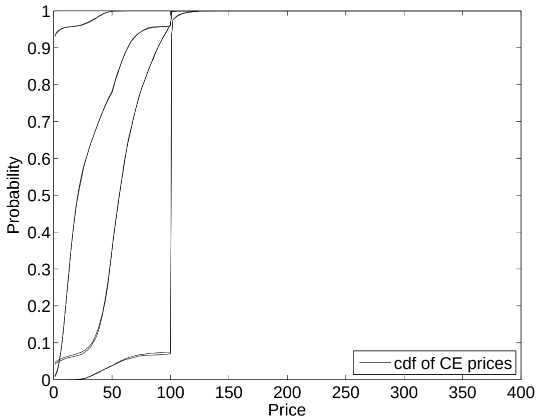
Figure 5: Confidence intervals of score mean, experiment with CE

### 3.4.3 Discussion

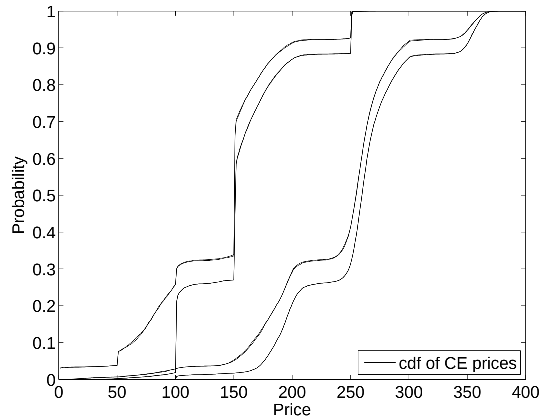
In both experiments, the mean of prediction is higher than that of the clearing prices and the standard deviations of prediction is lower than that of the clearing prices. I think the quality of the prediction is pretty good, although it is overpredicted by about 20. As we have seen from the previous experiments with over-prediction, it would just make us harder to see the order of the algorithms. Still, if there is a score difference between algorithms, the order between them will be preserved even with more accurate prediction methods.

We can compare the second experiment with the experiment with $\mu = (150, 150, 150, 150, 250, 250, 250, 250)$ and $\sigma = 60$, because its mean and standard deviation of clearing prices are similar to those of this experiment. The general order is similar, but TargetBidder and BidEvaluator is placed on a higher order in the game-theoretic setting. The second experiment also shows results similar to those of the first experiment.

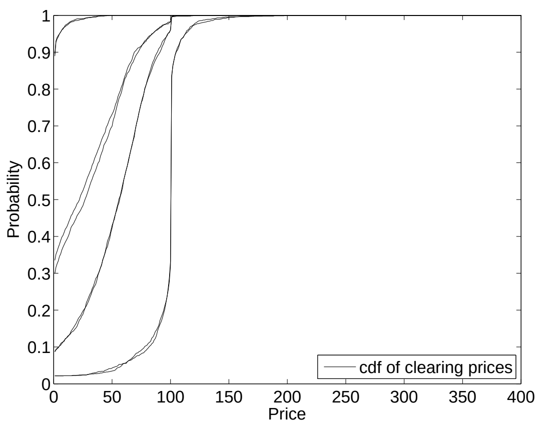
The results show that in the first experiment, it is hard to measure the performances among SAATop, BidEvaluator* and TargetBidder*. This is consistent with the fact that some MV-based algorithms performed as well as SAA-based algorithms in the TAC games. Still, as we can see in the second experiments, there is a case that MV-based algorithms performs worse than SAA-based algorithms. Note that in Figure 5(b), the performances



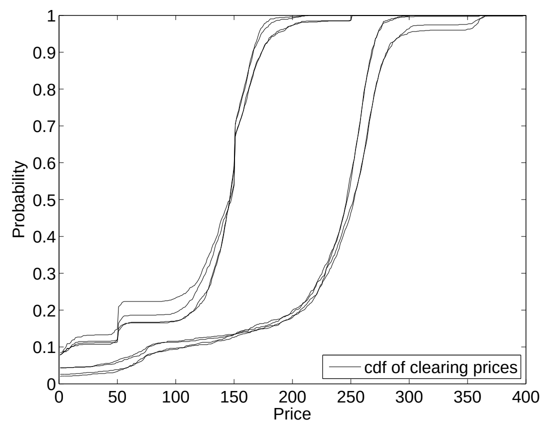

(a) Cdf of CE, 8 agents




(b) Cdf of CE, B(32,0.5) agents




(c) Cdf of clearing prices, 8 agents




(d) Cdf of clearing prices, B(32,0.5) agents

Figure 6: Cdf of CE prices and clearing prices, experiment with CE

of TargetBidder, BidEvaluator and BidEvaluator* might seem to be similar to those of SAA-based algorithms, but in reality, the formers performs strictly worse than the latter: the extremely low scores of AverageMU and StraightMU make the formers look better.

# 4 Conclusion

In this thesis, I tried to compare SAA-based algorithms and MV-based algorithms in a simultaneous auction. In general, SAA-based algorithms perform better than the others. First, they are optimal in a decision-theoretic setting with infinite number of scenarios. Second, in a decision-theoretic setting, they are more tolerant to variance. Third, in a decision-theoretic setting, they are more tolerant to noise, especially for SAATop. Additionaly, we found that SAABottom performs as well as SAATop in high variance settings. Finally, SAA-based algorithms showed a better performance even in the game-theoretic setting we illustrated.

However, we saw that BidEvaluator* and TargetBidder* perform as well as SAATop when there is a small variance on the distribution of clearing prices. With a small variance, the bidding problem with the clearing price distribution can be approximated to the bidding problem with a single clearing price, in which setting both algorithms are optimal.

Therefore, I suggest using a sample average approximation-based algorithm rather than using a marginal value-based algorithm for bidding problems. Although the TAC showed some MV-based algorithms perform best, there are cases that MV-based algorithms even fails to make profit. On the contrary, SAA-based algorithms perfom optimally when the prediction quality is good and the resource is enough.

## References

[CLL+04] S.F. Cheng, E. Leung, K.M. Lochner, K.O'Malley, D.M. Reeves, L.J. Schvartzman, and M.P. Wellman. Walverine: A Walrasian trading agent. *Decision Support Systems*, page To Appear, 2004.

[GB01] Amy Greenwald and Justin Boyan. Bidding algorithms for simultaneous auctions. In *EC '01: Proceedings of the 3rd ACM conference on Electronic Commerce*, pages 115–124, New York, NY, USA, 2001. ACM Press.

[GN06] Amy Greenwald and Victor Naroditskiy. Heuristics for the deterministic bidding problem. *SIGecom Exch.*, 6(1):35–44, 2006.

[LGN07] Seong Jae Lee, Amy R. Greenwald, and Victor Naroditskiy. Roxybot-06: An (saa)2 tac travel agent. In *IJCAI*, pages 1378–1383, 2007.

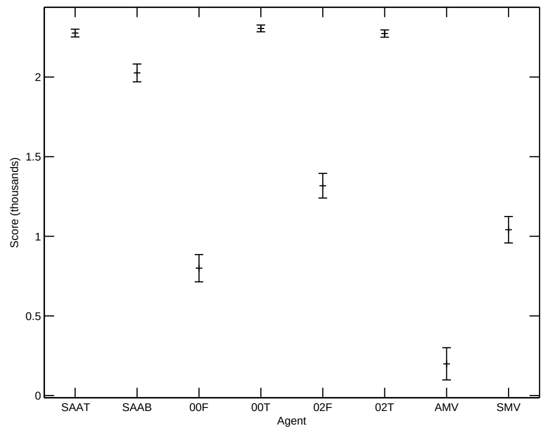
[OS06] Francisco Oliveira and Yain-Whar Si. Strategic issues in trading agent competition: Tac-classic. *wi-iatw*, 0:518–522, 2006.

[SLSK01] P. Stone, M.L. Littman, S. Singh, and M. Kearns. `ATTac`-2000: An adaptive autonomous bidding agent. In *Fifth International Conference on Autonomous Agents*, 2001.

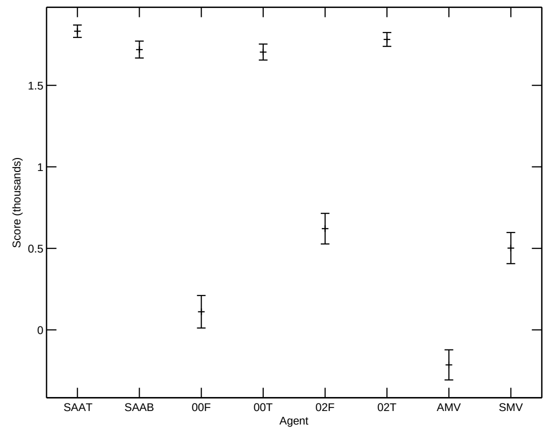
[VAK+03] Bram Verweij, Shabbir Ahmed, Anton J. Kleywegt, George Nemhauser, and Alexander Shapiro. The sample average approximation method applied to stochastic routing problems: A computational study. *Comput. Optim. Appl.*, 24(2-3):289–333, 2003.

# Appendix

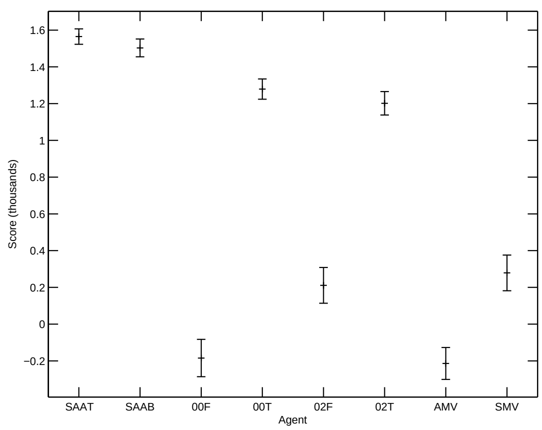
## A. Experiment with Perfect Prediction



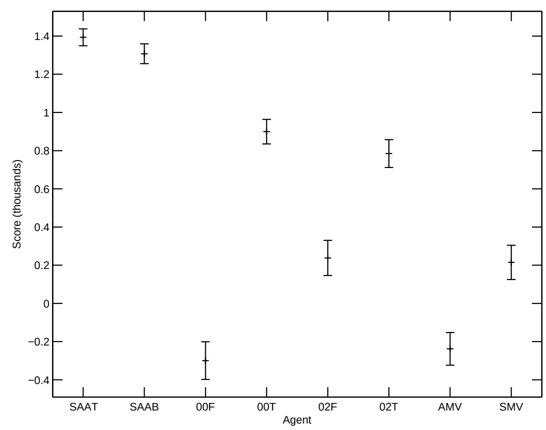

(a) $\lambda = 1$




(b) $\lambda = 2$

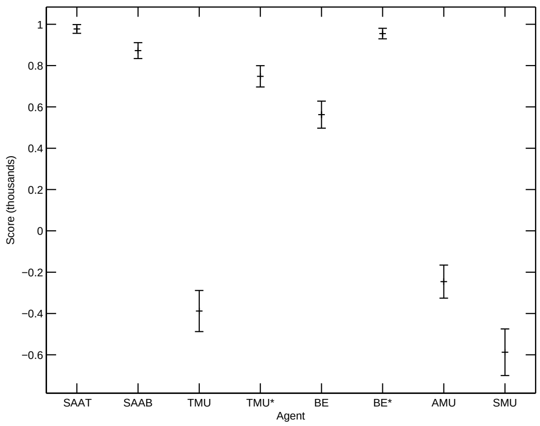



(c) $\lambda = 3$

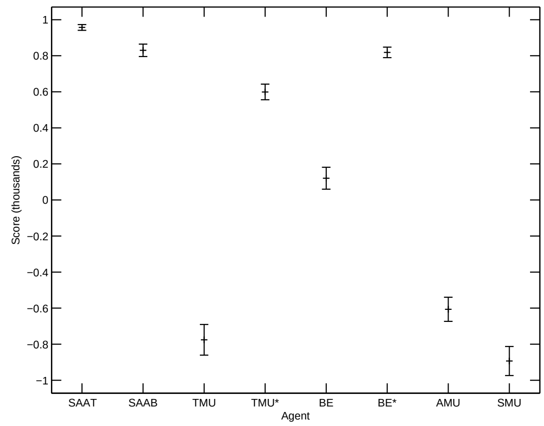



(d) $\lambda = 4$

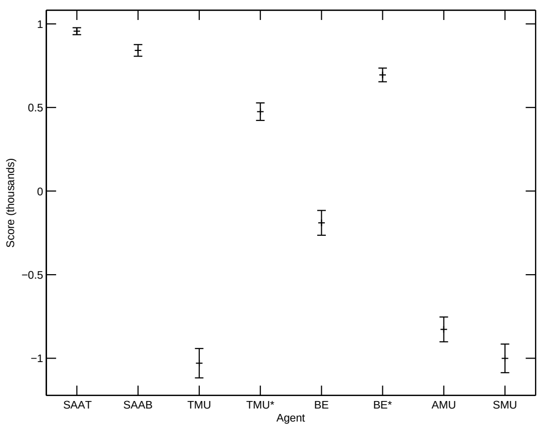
Figure 7: Confidence intervals of score mean, experiment with low mean



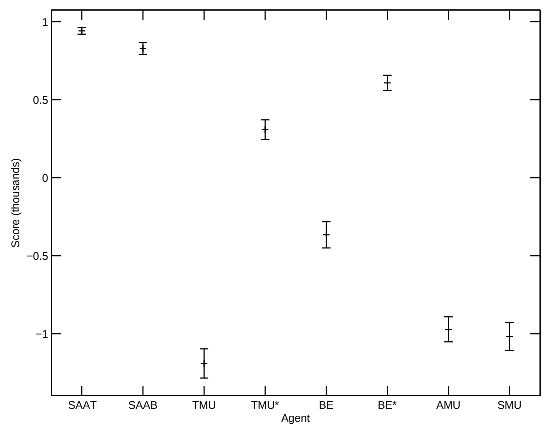

(a) $\sigma = 1$



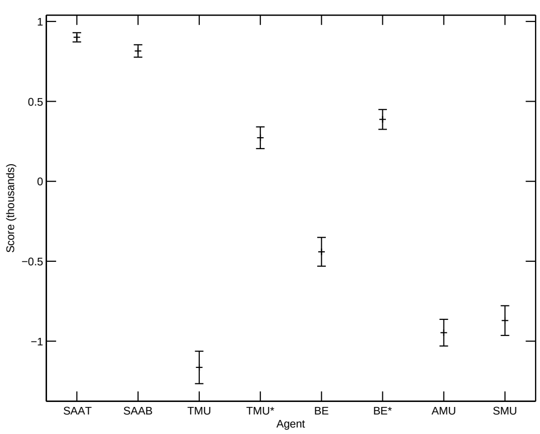

(b) $\sigma = 20$



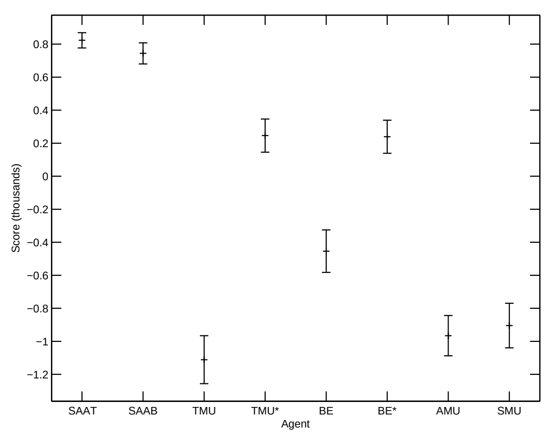

(c) $\sigma = 40$




(d) $\sigma = 60$

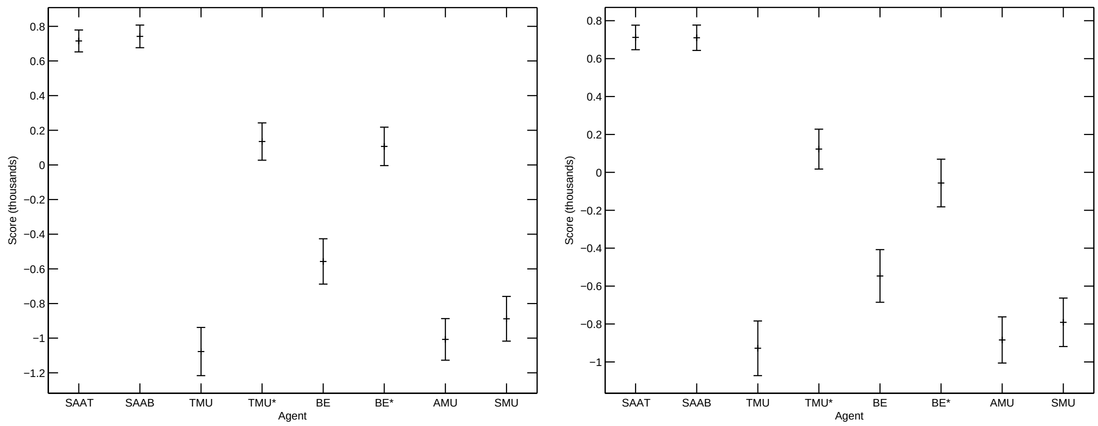



(e) $\sigma = 80$




(f) $\sigma = 100$

Figure 8: Confidence intervals of score mean, experiment with high mean I




(a) $\sigma = 120$ (b) $\sigma = 140$

Figure 9: Confidence intervals of score mean, experiment with high mean II

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 2276 | 2026 | 799 | 2305 | 1317 | 2272 | 199 | 1041 |
| Utility | 8220 | 7939 | 6464 | 8265 | 6996 | 8233 | 5821 | 6849 |
| Cost | 5944 | 5913 | 5664 | 5959 | 5678 | 5960 | 5622 | 5807 |
| Penalty | 282 | 268 | 244 | 275 | 375 | 274 | 77 | 247 |
| Null | 0.04 | 0.31 | 1.73 | 0.01 | 1.09 | 0.02 | 2.48 | 1.38 |
| F.unuse | 0.09 | 0.62 | 3.47 | 0.02 | 2.18 | 0.04 | 4.57 | 2.75 |
| F.use | 15.9 | 15.4 | 12.5 | 16.0 | 13.8 | 16.0 | 11.0 | 13.2 |
| F.cost | 5200 | 5200 | 5200 | 5200 | 5200 | 5200 | 5072 | 5200 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 547 | 520 | 443 | 553 | 460 | 529 | 384 | 474 |
| H.earn | 13.7 | 13.4 | 10.8 | 13.1 | 10.4 | 13.2 | 14.9 | 14.6 |
| H.unuse | 0.7 | 0.9 | 1.6 | 0.0 | 0.8 | 0.0 | 6.2 | 4.6 |
| H.use | 13.1 | 12.5 | 9.2 | 13.1 | 9.5 | 13.1 | 8.7 | 10.1 |
| H.cost | 744 | 713 | 464 | 759 | 478 | 760 | 550 | 607 |
| H.aver | 54.2 | 53.2 | 42.9 | 58.1 | 46.2 | 57.8 | 36.8 | 41.5 |

Table 2: Experiment with low mean, $\lambda = 1$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1832 | 1719 | 111 | 1704 | 620 | 1781 | -214 | 501 |
| Utility | 7822 | 7652 | 5727 | 7702 | 6218 | 7852 | 5165 | 6399 |
| Cost | 5990 | 5933 | 5616 | 5998 | 5597 | 6071 | 5380 | 5897 |
| Penalty | 558 | 568 | 380 | 534 | 525 | 453 | 174 | 416 |
| Null | 0.16 | 0.30 | 2.29 | 0.30 | 1.61 | 0.15 | 3.00 | 1.67 |
| F.unuse | 0.32 | 0.61 | 4.59 | 0.60 | 3.21 | 0.30 | 4.97 | 3.35 |
| F.use | 15.7 | 15.4 | 11.4 | 15.4 | 12.8 | 15.7 | 10.0 | 12.7 |
| F.cost | 5200 | 5200 | 5200 | 5200 | 5200 | 5200 | 4866 | 5200 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 539 | 526 | 403 | 537 | 349 | 455 | 338 | 490 |
| H.earn | 12.1 | 11.6 | 7.8 | 10.2 | 7.5 | 11.3 | 13.0 | 13.7 |
| H.unuse | 2.1 | 2.0 | 1.1 | 0.3 | 0.5 | 0.1 | 6.6 | 6.1 |
| H.use | 9.9 | 9.6 | 6.7 | 10.0 | 7.1 | 11.2 | 6.4 | 7.6 |
| H.cost | 790 | 733 | 416 | 798 | 397 | 871 | 514 | 697 |
| H.aver | 65.5 | 63.1 | 53.1 | 78.1 | 52.7 | 77.2 | 39.6 | 50.9 |

Table 3: Experiment with low mean, $\lambda = 2$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1564 | 1503 | -184 | 1278 | 211 | 1201 | -214 | 278 |
| Utility | 7489 | 7390 | 5373 | 7162 | 5755 | 7144 | 4994 | 6259 |
| Cost | 5924 | 5887 | 5557 | 5883 | 5544 | 5942 | 5208 | 5980 |
| Penalty | 779 | 751 | 461 | 676 | 613 | 683 | 255 | 574 |
| Null | 0.20 | 0.29 | 2.46 | 0.57 | 1.85 | 0.57 | 3.07 | 1.63 |
| F.unuse | 0.40 | 0.58 | 4.93 | 1.15 | 3.67 | 1.13 | 4.66 | 3.27 |
| F.use | 15.6 | 15.4 | 11.1 | 14.9 | 12.3 | 14.9 | 9.9 | 12.7 |
| F.cost | 5200 | 5200 | 5200 | 5200 | 5193 | 5199 | 4719 | 5201 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 470 | 431 | 300 | 412 | 215 | 396 | 319 | 469 |
| H.earn | 10.7 | 10.3 | 5.8 | 7.7 | 6.5 | 8.5 | 11.5 | 13.4 |
| H.unuse | 2.9 | 2.5 | 0.2 | 0.1 | 0.2 | 0.2 | 6.1 | 6.9 |
| H.use | 7.8 | 7.8 | 5.6 | 7.6 | 6.3 | 8.3 | 5.5 | 6.5 |
| H.cost | 724 | 687 | 357 | 683 | 350 | 744 | 489 | 779 |
| H.aver | 67.7 | 66.7 | 62.0 | 88.8 | 54.1 | 87.5 | 42.4 | 58.2 |

Table 4: Experiment with low mean, $\lambda = 3$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1393 | 1307 | -299 | 899 | 238 | 784 | -238 | 214 |
| Utility | 7375 | 7244 | 5237 | 6702 | 5729 | 6504 | 4855 | 6280 |
| Cost | 5981 | 5937 | 5537 | 5802 | 5491 | 5719 | 5093 | 6066 |
| Penalty | 782 | 786 | 486 | 655 | 673 | 765 | 307 | 603 |
| Null | 0.21 | 0.31 | 2.47 | 0.91 | 1.72 | 0.97 | 3.13 | 1.54 |
| F.unuse | 0.42 | 0.61 | 4.94 | 1.81 | 3.32 | 1.82 | 4.36 | 3.09 |
| F.use | 15.6 | 15.4 | 11.1 | 14.2 | 12.6 | 14.1 | 9.7 | 12.9 |
| F.cost | 5200 | 5200 | 5200 | 5200 | 5160 | 5164 | 4581 | 5200 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 365 | 339 | 193 | 264 | 124 | 235 | 295 | 427 |
| H.earn | 10.6 | 10.2 | 5.5 | 7.1 | 6.4 | 7.2 | 11.0 | 13.9 |
| H.unuse | 2.9 | 2.5 | 0.0 | 0.0 | 0.1 | 0.0 | 5.9 | 7.5 |
| H.use | 7.8 | 7.7 | 5.5 | 7.1 | 6.3 | 7.2 | 5.2 | 6.5 |
| H.cost | 781 | 737 | 337 | 602 | 331 | 555 | 512 | 866 |
| H.aver | 73.4 | 72.1 | 61.0 | 84.9 | 52.0 | 77.0 | 46.4 | 62.2 |

Table 5: Experiment with low mean, $\lambda = 4$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 977 | 872 | -388 | 748 | 562 | 955 | -245 | -587 |
| Utility | 7139 | 6594 | 5250 | 6725 | 6105 | 6488 | 5409 | 5676 |
| Cost | 6162 | 5722 | 5638 | 5977 | 5542 | 5533 | 5655 | 6264 |
| Penalty | 564 | 450 | 254 | 454 | 352 | 396 | 254 | 305 |
| Null | 0.75 | 1.40 | 2.82 | 1.26 | 1.96 | 1.57 | 2.70 | 2.40 |
| F.unuse | 0.00 | 0.26 | 3.80 | 0.72 | 1.16 | 0.08 | 2.56 | 3.18 |
| F.use | 14.5 | 13.2 | 10.4 | 13.5 | 12.1 | 12.9 | 10.6 | 11.2 |
| F.cost | 4713 | 4375 | 4603 | 4616 | 4300 | 4204 | 4276 | 4675 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 454 | 447 | 322 | 439 | 422 | 458 | 365 | 382 |
| H.earn | 7.3 | 6.6 | 5.2 | 6.7 | 6.0 | 6.4 | 6.9 | 8.0 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.6 | 2.4 |
| H.use | 7.3 | 6.6 | 5.2 | 6.7 | 6.0 | 6.4 | 5.3 | 5.6 |
| H.cost | 1449 | 1347 | 1035 | 1360 | 1242 | 1329 | 1380 | 1589 |
| H.aver | 200.0 | 204.3 | 199.8 | 201.9 | 205.8 | 206.9 | 200.1 | 199.5 |

Table 6: Experiment with high mean, $\sigma = 1$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 957 | 830 | -775 | 599 | 120 | 818 | -606 | -893 |
| Utility | 6997 | 6804 | 4708 | 6472 | 5605 | 6351 | 4788 | 5413 |
| Cost | 6040 | 5974 | 5484 | 5872 | 5484 | 5533 | 5394 | 6306 |
| Penalty | 549 | 512 | 250 | 430 | 307 | 381 | 208 | 322 |
| Null | 0.90 | 1.12 | 3.34 | 1.53 | 2.46 | 1.71 | 3.33 | 2.64 |
| F.unuse | 0.01 | 0.38 | 4.80 | 1.11 | 2.40 | 0.44 | 3.50 | 3.51 |
| F.use | 14.2 | 13.8 | 9.3 | 12.9 | 11.1 | 12.6 | 9.3 | 10.7 |
| F.cost | 4620 | 4594 | 4590 | 4570 | 4378 | 4233 | 4171 | 4627 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 445 | 441 | 298 | 429 | 376 | 442 | 329 | 372 |
| H.earn | 7.1 | 6.9 | 4.7 | 6.5 | 5.5 | 6.3 | 6.6 | 9.1 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.9 | 3.7 |
| H.use | 7.1 | 6.9 | 4.7 | 6.5 | 5.5 | 6.3 | 4.7 | 5.4 |
| H.cost | 1421 | 1380 | 894 | 1302 | 1106 | 1300 | 1223 | 1679 |
| H.aver | 200.0 | 200.7 | 192.0 | 201.2 | 199.8 | 206.7 | 185.5 | 184.5 |

Table 7: Experiment with high mean, $\sigma = 20$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 956 | 841 | -1029 | 474 | -190 | 694 | -827 | -1000 |
| Utility | 7024 | 6820 | 4328 | 6346 | 5251 | 6320 | 4350 | 5370 |
| Cost | 6068 | 5978 | 5357 | 5871 | 5441 | 5625 | 5177 | 6371 |
| Penalty | 541 | 521 | 248 | 387 | 285 | 366 | 163 | 349 |
| Null | 0.89 | 1.09 | 3.69 | 1.66 | 2.79 | 1.73 | 3.77 | 2.66 |
| F.unuse | 0.04 | 0.38 | 5.53 | 1.60 | 3.28 | 0.86 | 4.10 | 3.56 |
| F.use | 14.2 | 13.8 | 8.6 | 12.7 | 10.4 | 12.5 | 8.4 | 10.7 |
| F.cost | 4637 | 4618 | 4597 | 4640 | 4454 | 4358 | 4080 | 4629 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 452 | 429 | 269 | 397 | 323 | 414 | 289 | 380 |
| H.earn | 7.1 | 6.9 | 4.3 | 6.3 | 5.2 | 6.3 | 6.8 | 10.5 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.5 | 5.2 |
| H.use | 7.1 | 6.9 | 4.3 | 6.3 | 5.2 | 6.3 | 4.3 | 5.3 |
| H.cost | 1431 | 1360 | 761 | 1231 | 987 | 1268 | 1097 | 1742 |
| H.aver | 200.6 | 196.6 | 176.7 | 194.4 | 189.3 | 201.8 | 160.8 | 165.4 |

Table 8: Experiment with high mean, $\sigma = 40$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 941 | 829 | -1190 | 308 | -365 | 608 | -971 | -1017 |
| Utility | 6985 | 6801 | 4064 | 6053 | 5011 | 6225 | 4065 | 5202 |
| Cost | 6044 | 5972 | 5255 | 5745 | 5377 | 5617 | 5037 | 6220 |
| Penalty | 535 | 520 | 234 | 357 | 267 | 352 | 136 | 341 |
| Null | 0.93 | 1.12 | 3.96 | 1.97 | 3.02 | 1.82 | 4.06 | 2.82 |
| F.unuse | 0.03 | 0.39 | 6.05 | 2.01 | 3.82 | 1.08 | 4.63 | 3.76 |
| F.use | 14.1 | 13.8 | 8.1 | 12.1 | 10.0 | 12.4 | 7.9 | 10.3 |
| F.cost | 4604 | 4601 | 4594 | 4571 | 4475 | 4367 | 4062 | 4587 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 454 | 440 | 258 | 385 | 303 | 400 | 267 | 369 |
| H.earn | 7.1 | 6.9 | 4.0 | 6.0 | 5.0 | 6.2 | 7.1 | 11.0 |
| H.unuse | 0.1 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 3.0 | 5.8 |
| H.use | 7.1 | 6.9 | 4.0 | 6.0 | 5.0 | 6.2 | 4.0 | 5.2 |
| H.cost | 1440 | 1371 | 661 | 1174 | 901 | 1249 | 975 | 1633 |
| H.aver | 202.1 | 197.9 | 163.7 | 194.8 | 181.3 | 201.6 | 138.0 | 148.9 |

Table 9: Experiment with high mean, $\sigma = 60$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 901 | 815 | -1164 | 272 | -441 | 387 | -946 | -871 |
| Utility | 6912 | 6774 | 4014 | 5973 | 4879 | 5972 | 3928 | 5272 |
| Cost | 6011 | 5958 | 5178 | 5700 | 5320 | 5585 | 4875 | 6143 |
| Penalty | 507 | 501 | 226 | 330 | 254 | 339 | 129 | 349 |
| Null | 1.06 | 1.18 | 4.02 | 2.09 | 3.16 | 2.05 | 4.20 | 2.75 |
| F.unuse | 0.11 | 0.42 | 6.07 | 2.17 | 4.02 | 1.71 | 4.68 | 3.57 |
| F.use | 13.9 | 13.6 | 7.9 | 11.8 | 9.7 | 11.9 | 7.6 | 10.5 |
| F.cost | 4546 | 4567 | 4556 | 4546 | 4453 | 4420 | 3992 | 4573 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 481 | 459 | 266 | 396 | 292 | 367 | 256 | 373 |
| H.earn | 7.1 | 6.9 | 4.0 | 5.9 | 4.8 | 6.0 | 7.5 | 11.7 |
| H.unuse | 0.2 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 3.6 | 6.5 |
| H.use | 6.9 | 6.8 | 4.0 | 5.9 | 4.8 | 6.0 | 3.9 | 5.2 |
| H.cost | 1465 | 1391 | 623 | 1154 | 867 | 1165 | 883 | 1571 |
| H.aver | 205.4 | 201.0 | 156.8 | 195.4 | 179.2 | 194.4 | 118.1 | 133.8 |

Table 10: Experiment with high mean, $\sigma = 80$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 841 | 766 | -1107 | 252 | -430 | 263 | -924 | -868 |
| Utility | 6752 | 6644 | 3891 | 5779 | 4822 | 5760 | 3863 | 5061 |
| Cost | 5910 | 5878 | 4998 | 5527 | 5252 | 5496 | 4788 | 5929 |
| Penalty | 476 | 460 | 219 | 319 | 260 | 316 | 124 | 319 |
| Null | 1.29 | 1.39 | 4.18 | 2.33 | 3.22 | 2.29 | 4.27 | 2.99 |
| F.unuse | 0.21 | 0.45 | 5.93 | 2.21 | 4.04 | 2.01 | 4.74 | 3.53 |
| F.use | 13.4 | 13.2 | 7.6 | 11.3 | 9.6 | 11.4 | 7.5 | 10.0 |
| F.cost | 4431 | 4443 | 4410 | 4402 | 4421 | 4367 | 3963 | 4405 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 516 | 496 | 291 | 434 | 301 | 363 | 262 | 367 |
| H.earn | 7.1 | 6.8 | 3.8 | 5.7 | 4.8 | 5.8 | 7.7 | 12.2 |
| H.unuse | 0.4 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 3.8 | 7.2 |
| H.use | 6.7 | 6.6 | 3.8 | 5.7 | 4.8 | 5.7 | 3.8 | 5.0 |
| H.cost | 1479 | 1435 | 588 | 1125 | 831 | 1129 | 825 | 1524 |
| H.aver | 208.1 | 209.5 | 154.0 | 198.7 | 173.8 | 196.2 | 107.7 | 125.3 |

Table 11: Experiment with high mean, $\sigma = 100$

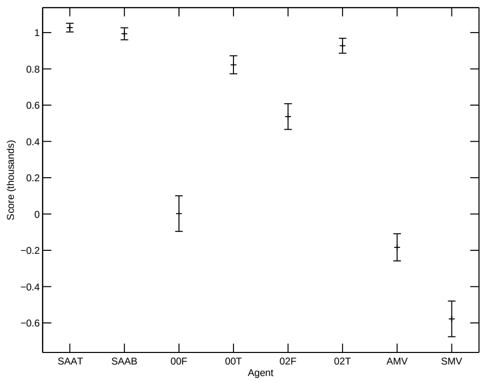
| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 720 | 733 | -1150 | 152 | -551 | 77 | -996 | -823 |
| Utility | 6489 | 6553 | 3686 | 5556 | 4604 | 5522 | 3688 | 4993 |
| Cost | 5768 | 5820 | 4836 | 5403 | 5156 | 5445 | 4684 | 5816 |
| Penalty | 435 | 432 | 199 | 275 | 246 | 301 | 111 | 293 |
| Null | 1.60 | 1.53 | 4.41 | 2.62 | 3.43 | 2.54 | 4.46 | 3.08 |
| F.unuse | 0.39 | 0.50 | 5.96 | 2.44 | 4.30 | 2.59 | 4.93 | 3.44 |
| F.use | 12.8 | 12.9 | 7.2 | 10.8 | 9.1 | 10.9 | 7.1 | 9.8 |
| F.cost | 4285 | 4364 | 4271 | 4291 | 4365 | 4392 | 3904 | 4313 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 526 | 520 | 296 | 449 | 284 | 360 | 260 | 373 |
| H.earn | 7.0 | 6.8 | 3.6 | 5.4 | 4.6 | 5.5 | 7.8 | 12.5 |
| H.unuse | 0.6 | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 | 4.1 | 7.6 |
| H.use | 6.4 | 6.5 | 3.6 | 5.4 | 4.6 | 5.5 | 3.7 | 4.9 |
| H.cost | 1483 | 1456 | 565 | 1112 | 791 | 1053 | 780 | 1504 |
| H.aver | 212.1 | 214.8 | 157.5 | 206.7 | 173.2 | 192.0 | 100.2 | 120.6 |

Table 12: Experiment with high mean, $\sigma = 120$

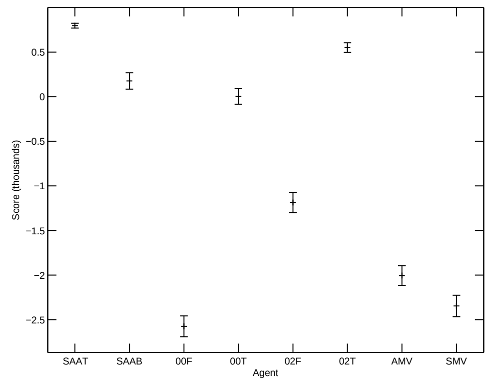
| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 711 | 710 | -928 | 122 | -546 | -56 | -884 | -791 |
| Utility | 6372 | 6371 | 3899 | 5392 | 4471 | 5351 | 3660 | 4955 |
| Cost | 5660 | 5661 | 4827 | 5269 | 5018 | 5407 | 4544 | 5746 |
| Penalty | 402 | 401 | 197 | 272 | 237 | 320 | 125 | 294 |
| Null | 1.76 | 1.76 | 4.23 | 2.79 | 3.57 | 2.71 | 4.48 | 3.13 |
| F.unuse | 0.45 | 0.50 | 5.50 | 2.53 | 4.25 | 2.96 | 4.57 | 3.22 |
| F.use | 12.5 | 12.5 | 7.5 | 10.4 | 8.9 | 10.6 | 7.0 | 9.7 |
| F.cost | 4200 | 4218 | 4235 | 4209 | 4259 | 4402 | 3772 | 4215 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 537 | 533 | 331 | 454 | 284 | 382 | 269 | 378 |
| H.earn | 6.9 | 6.7 | 3.8 | 5.2 | 4.4 | 5.3 | 8.1 | 13.0 |
| H.unuse | 0.7 | 0.4 | 0.0 | 0.0 | 0.0 | 0.0 | 4.5 | 8.1 |
| H.use | 6.2 | 6.2 | 3.8 | 5.2 | 4.4 | 5.3 | 3.6 | 4.9 |
| H.cost | 1459 | 1442 | 592 | 1060 | 759 | 1005 | 772 | 1531 |
| H.aver | 210.5 | 216.5 | 157.2 | 203.5 | 171.5 | 189.6 | 95.5 | 118.3 |

Table 13: Experiment with high mean, $\sigma = 140$

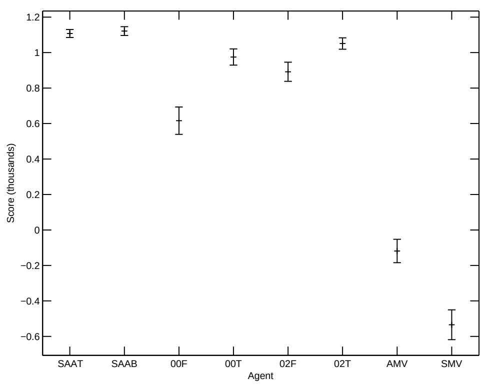
## B. Experiment with Noisy Prediction



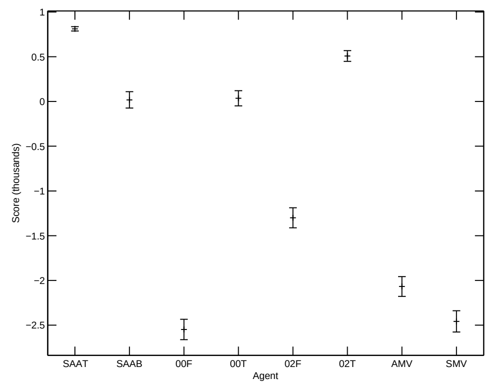

(a) $\lambda = -10$




(b) $\lambda = 10$

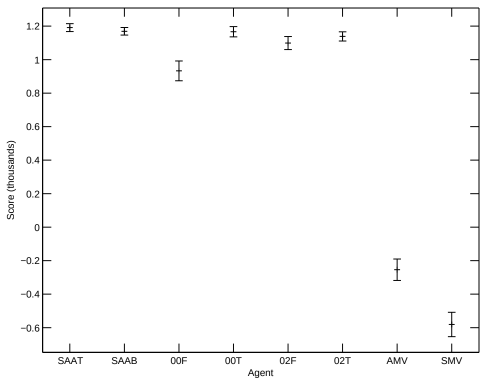



(c) $\lambda = -20$

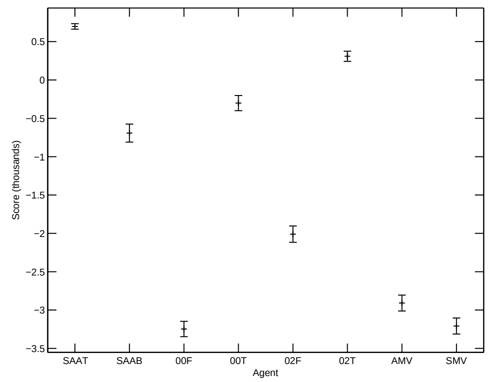



(d) $\lambda = 20$

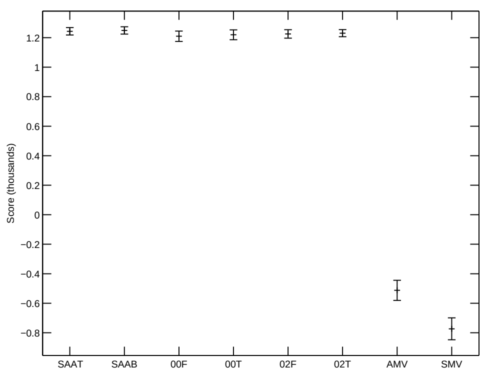
Figure 10: Confidence intervals of score mean, experiment with noise, $\sigma = 20$, I



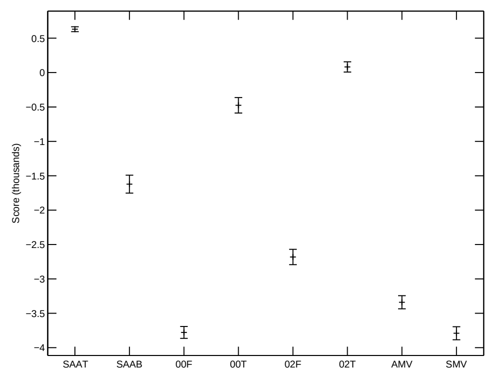

(a) $\lambda = -30$




(b) $\lambda = 30$




(c) $\lambda = -40$




(d) $\lambda = 40$

Figure 11: Confidence intervals of score mean, experiment with noise, $\sigma = 20$, II

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1243 | 1249 | 1209 | 1219 | 1225 | 1230 | -512 | -773 |
| Utility | 6997 | 7019 | 6976 | 6914 | 6682 | 6631 | 6747 | 7252 |
| Cost | 5754 | 5770 | 5767 | 5695 | 5457 | 5401 | 7259 | 8025 |
| Penalty | 537 | 542 | 530 | 527 | 443 | 449 | 419 | 533 |
| Null | 0.91 | 0.89 | 0.95 | 0.99 | 1.31 | 1.39 | 1.46 | 0.88 |
| F.unuse | 0.00 | 0.00 | 0.15 | 0.07 | 0.07 | 0.00 | 0.12 | 0.05 |
| F.use | 14.2 | 14.2 | 14.1 | 14.0 | 13.4 | 13.2 | 13.1 | 14.2 |
| F.cost | 4607 | 4619 | 4635 | 4580 | 4368 | 4298 | 4289 | 4644 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 448 | 456 | 453 | 433 | 440 | 469 | 626 | 666 |
| H.earn | 7.2 | 7.2 | 7.1 | 7.0 | 6.7 | 6.6 | 19.2 | 21.6 |
| H.unuse | 0.1 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 12.5 | 14.5 |
| H.use | 7.1 | 7.1 | 7.1 | 7.0 | 6.7 | 6.6 | 6.7 | 7.1 |
| H.cost | 1147 | 1151 | 1132 | 1115 | 1089 | 1103 | 2970 | 3382 |
| H.aver | 159.9 | 160.0 | 160.6 | 159.1 | 162.9 | 166.9 | 154.8 | 156.7 |

Table 14: Experiment with noise, $\sigma = 20$, $\lambda = -40$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1191 | 1169 | 932 | 1166 | 1098 | 1138 | -254 | -580 |
| Utility | 7075 | 6925 | 6647 | 7009 | 6699 | 6588 | 6612 | 7147 |
| Cost | 5884 | 5756 | 5714 | 5843 | 5600 | 5449 | 6866 | 7728 |
| Penalty | 541 | 527 | 506 | 514 | 460 | 436 | 398 | 526 |
| Null | 0.83 | 1.00 | 1.27 | 0.93 | 1.29 | 1.43 | 1.57 | 0.96 |
| F.unuse | 0.00 | 0.00 | 0.65 | 0.12 | 0.24 | 0.06 | 0.24 | 0.14 |
| F.use | 14.3 | 14.0 | 13.5 | 14.1 | 13.4 | 13.1 | 12.9 | 14.1 |
| F.cost | 4659 | 4553 | 4588 | 4633 | 4440 | 4289 | 4258 | 4625 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 449 | 449 | 422 | 456 | 448 | 456 | 578 | 629 |
| H.earn | 7.3 | 7.1 | 6.7 | 7.1 | 6.7 | 6.6 | 16.1 | 18.9 |
| H.unuse | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 9.5 | 11.8 |
| H.use | 7.2 | 7.0 | 6.7 | 7.1 | 6.7 | 6.6 | 6.5 | 7.0 |
| H.cost | 1225 | 1203 | 1126 | 1210 | 1161 | 1161 | 2608 | 3102 |
| H.aver | 168.9 | 170.8 | 167.4 | 171.2 | 173.0 | 176.7 | 162.3 | 164.4 |

Table 15: Experiment with noise, $\sigma = 20$, $\lambda = -30$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1107 | 1121 | 616 | 974 | 891 | 1051 | -118 | -534 |
| Utility | 7048 | 7056 | 6369 | 6868 | 6462 | 6541 | 6346 | 6822 |
| Cost | 5941 | 5935 | 5752 | 5893 | 5570 | 5490 | 6464 | 7356 |
| Penalty | 548 | 532 | 443 | 519 | 399 | 417 | 369 | 497 |
| Null | 0.85 | 0.87 | 1.59 | 1.06 | 1.57 | 1.50 | 1.80 | 1.25 |
| F.unuse | 0.00 | 0.02 | 1.39 | 0.40 | 0.67 | 0.14 | 0.74 | 0.60 |
| F.use | 14.3 | 14.3 | 12.8 | 13.9 | 12.9 | 13.0 | 12.4 | 13.5 |
| F.cost | 4648 | 4641 | 4622 | 4642 | 4394 | 4271 | 4272 | 4584 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 447 | 457 | 399 | 443 | 435 | 459 | 513 | 566 |
| H.earn | 7.2 | 7.2 | 6.4 | 6.9 | 6.4 | 6.5 | 12.9 | 16.1 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 6.6 | 9.4 |
| H.use | 7.2 | 7.1 | 6.4 | 6.9 | 6.4 | 6.5 | 6.3 | 6.8 |
| H.cost | 1293 | 1294 | 1131 | 1251 | 1176 | 1219 | 2192 | 2773 |
| H.aver | 179.7 | 180.8 | 176.4 | 180.2 | 183.1 | 187.7 | 170.4 | 171.7 |

Table 16: Experiment with noise, $\sigma = 20$, $\lambda = -20$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 1027 | 993 | 2 | 822 | 536 | 927 | -183 | -577 |
| Utility | 7018 | 6933 | 5619 | 6744 | 6016 | 6490 | 5798 | 6295 |
| Cost | 5991 | 5940 | 5617 | 5922 | 5479 | 5562 | 5982 | 6873 |
| Penalty | 553 | 524 | 356 | 476 | 362 | 418 | 301 | 420 |
| Null | 0.86 | 0.98 | 2.39 | 1.22 | 2.01 | 1.56 | 2.33 | 1.77 |
| F.unuse | 0.00 | 0.12 | 2.85 | 0.67 | 1.39 | 0.29 | 1.82 | 1.73 |
| F.use | 14.3 | 14.0 | 11.2 | 13.6 | 12.0 | 12.9 | 11.3 | 12.5 |
| F.cost | 4638 | 4599 | 4576 | 4628 | 4346 | 4281 | 4277 | 4614 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 436 | 440 | 362 | 437 | 388 | 466 | 429 | 482 |
| H.earn | 7.2 | 7.0 | 5.6 | 6.8 | 6.0 | 6.4 | 9.6 | 12.6 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3.9 | 6.4 |
| H.use | 7.1 | 7.0 | 5.6 | 6.8 | 6.0 | 6.4 | 5.7 | 6.2 |
| H.cost | 1353 | 1340 | 1041 | 1294 | 1134 | 1281 | 1705 | 2260 |
| H.aver | 189.0 | 190.6 | 185.5 | 190.8 | 189.3 | 199.0 | 177.7 | 179.2 |

Table 17: Experiment with noise, $\sigma = 20$, $\lambda = -10$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 796 | 176 | -2574 | 2 | -1186 | 550 | -2004 | -2346 |
| Utility | 6932 | 6144 | 2613 | 5916 | 4010 | 6146 | 2803 | 3087 |
| Cost | 6136 | 5967 | 5188 | 5914 | 5197 | 5595 | 4808 | 5433 |
| Penalty | 539 | 430 | 67 | 311 | 156 | 327 | 58 | 118 |
| Null | 0.97 | 1.83 | 5.49 | 2.16 | 4.09 | 1.96 | 5.32 | 4.99 |
| F.unuse | 0.08 | 1.88 | 9.34 | 2.56 | 5.61 | 0.92 | 7.55 | 8.38 |
| F.use | 14.1 | 12.3 | 5.0 | 11.7 | 7.8 | 12.1 | 5.4 | 6.0 |
| F.cost | 4594 | 4622 | 4668 | 4625 | 4365 | 4229 | 4197 | 4680 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 442 | 406 | 169 | 392 | 257 | 429 | 180 | 196 |
| H.earn | 7.0 | 6.2 | 2.5 | 5.8 | 3.9 | 6.0 | 3.0 | 3.8 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.3 | 0.8 |
| H.use | 7.0 | 6.2 | 2.5 | 5.8 | 3.9 | 6.0 | 2.7 | 3.0 |
| H.cost | 1541 | 1345 | 519 | 1288 | 831 | 1366 | 611 | 753 |
| H.aver | 219.3 | 218.2 | 207.0 | 220.8 | 212.7 | 226.1 | 203.3 | 198.6 |

Table 18: Experiment with noise, $\sigma = 20$, $\lambda = 10$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 811 | 17 | -2548 | 35 | -1299 | 507 | -2067 | -2457 |
| Utility | 6980 | 5913 | 2606 | 5965 | 3903 | 6183 | 2728 | 2898 |
| Cost | 6169 | 5895 | 5154 | 5929 | 5203 | 5675 | 4796 | 5356 |
| Penalty | 526 | 439 | 74 | 322 | 153 | 327 | 52 | 98 |
| Null | 0.93 | 2.01 | 5.48 | 2.10 | 4.20 | 1.91 | 5.39 | 5.18 |
| F.unuse | 0.04 | 2.20 | 9.23 | 2.43 | 5.90 | 1.05 | 7.75 | 8.60 |
| F.use | 14.1 | 12.0 | 5.0 | 11.8 | 7.6 | 12.2 | 5.2 | 5.6 |
| F.cost | 4611 | 4609 | 4638 | 4623 | 4390 | 4302 | 4215 | 4625 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 435 | 363 | 161 | 392 | 253 | 419 | 174 | 180 |
| H.earn | 7.1 | 6.0 | 2.5 | 5.9 | 3.8 | 6.1 | 2.9 | 3.7 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.2 | 0.9 |
| H.use | 7.1 | 6.0 | 2.5 | 5.9 | 3.8 | 6.1 | 2.6 | 2.8 |
| H.cost | 1558 | 1287 | 515 | 1307 | 813 | 1373 | 581 | 730 |
| H.aver | 220.3 | 214.9 | 204.8 | 221.7 | 213.8 | 225.5 | 204.1 | 198.5 |

Table 19: Experiment with noise, $\sigma = 20$, $\lambda = 20$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 698 | -692 | -3247 | -301 | -2010 | 309 | -2908 | -3208 |
| Utility | 6941 | 5026 | 1680 | 5633 | 2988 | 5952 | 1703 | 1811 |
| Cost | 6242 | 5719 | 4927 | 5934 | 4999 | 5643 | 4612 | 5020 |
| Penalty | 532 | 343 | 24 | 256 | 85 | 281 | 23 | 41 |
| Null | 0.97 | 2.94 | 6.40 | 2.46 | 5.12 | 2.15 | 6.38 | 6.26 |
| F.unuse | 0.15 | 4.02 | 10.92 | 3.27 | 7.66 | 1.46 | 9.86 | 10.69 |
| F.use | 14.1 | 10.1 | 3.2 | 11.1 | 5.8 | 11.7 | 3.2 | 3.5 |
| F.cost | 4618 | 4596 | 4588 | 4662 | 4362 | 4277 | 4256 | 4605 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 446 | 312 | 107 | 352 | 193 | 386 | 109 | 113 |
| H.earn | 7.0 | 5.1 | 1.6 | 5.5 | 2.9 | 5.8 | 1.7 | 2.0 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.3 |
| H.use | 7.0 | 5.1 | 1.6 | 5.5 | 2.9 | 5.8 | 1.6 | 1.7 |
| H.cost | 1625 | 1124 | 339 | 1272 | 637 | 1366 | 356 | 415 |
| H.aver | 231.3 | 222.3 | 212.7 | 229.8 | 221.2 | 233.7 | 210.8 | 207.6 |

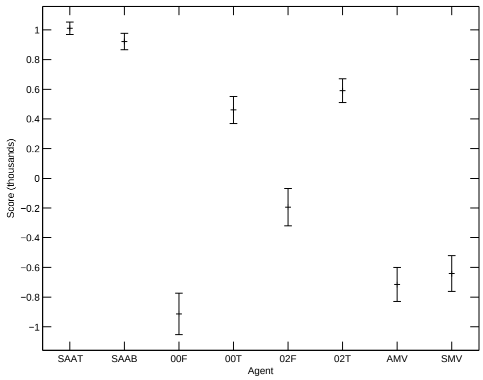
Table 20: Experiment with noise, $\sigma = 20$, $\lambda = 30$

| Agent | SAAT | SAAB | 00F | 00T | 02F | 02T | AMV | SMV |
|---|---|---|---|---|---|---|---|---|
| Score | 630 | -1622 | -3779 | -476 | -2681 | 82 | -3340 | -3791 |
| Utility | 6966 | 3884 | 1068 | 5420 | 2177 | 5711 | 1074 | 1109 |
| Cost | 6336 | 5506 | 4847 | 5896 | 4859 | 5628 | 4414 | 4900 |
| Penalty | 530 | 233 | 6 | 230 | 46 | 266 | 8 | 17 |
| Null | 0.95 | 4.13 | 6.99 | 2.68 | 5.92 | 2.42 | 6.99 | 6.95 |
| F.unuse | 0.17 | 6.47 | 12.22 | 3.61 | 9.32 | 1.90 | 10.86 | 12.21 |
| F.use | 14.1 | 7.7 | 2.0 | 10.6 | 4.2 | 11.2 | 2.0 | 2.1 |
| F.cost | 4638 | 4620 | 4627 | 4628 | 4385 | 4248 | 4190 | 4653 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 448 | 244 | 67 | 335 | 140 | 394 | 69 | 73 |
| H.earn | 7.1 | 3.9 | 1.0 | 5.3 | 2.1 | 5.6 | 1.0 | 1.1 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 |
| H.use | 7.1 | 3.9 | 1.0 | 5.3 | 2.1 | 5.6 | 1.0 | 1.1 |
| H.cost | 1697 | 886 | 220 | 1268 | 474 | 1380 | 224 | 247 |
| H.aver | 240.8 | 228.8 | 219.1 | 238.6 | 227.6 | 247.2 | 219.0 | 217.5 |

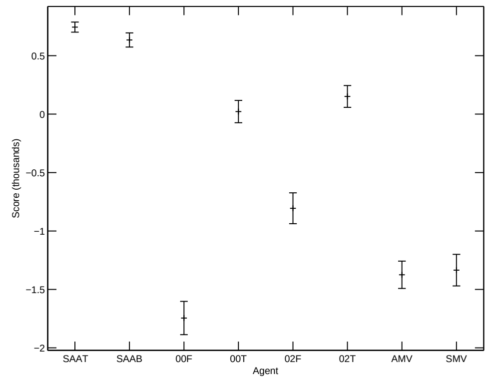
Table 21: Experiment with noise, $\sigma = 20$, $\lambda = 40$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 1186 | 1176 | -115 | 857 | 337 | 812 | -236 | -332 |
| Utility | 6946 | 6956 | 5115 | 6426 | 5645 | 6265 | 4977 | 6263 |
| Cost | 5759 | 5780 | 5231 | 5568 | 5308 | 5453 | 5214 | 6595 |
| Penalty | 507 | 524 | 332 | 409 | 340 | 395 | 182 | 443 |
| Null | 1.06 | 1.01 | 2.90 | 1.60 | 2.35 | 1.74 | 3.21 | 1.78 |
| F.unuse | 0.02 | 0.08 | 3.71 | 1.16 | 2.39 | 1.04 | 2.66 | 1.52 |
| F.use | 13.9 | 14.0 | 10.2 | 12.8 | 11.3 | 12.5 | 9.6 | 12.4 |
| F.cost | 4514 | 4568 | 4518 | 4534 | 4447 | 4408 | 3981 | 4534 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 518 | 491 | 352 | 437 | 340 | 399 | 367 | 491 |
| H.earn | 7.3 | 7.3 | 5.1 | 6.4 | 5.6 | 6.3 | 11.8 | 17.2 |
| H.unuse | 0.4 | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 | 6.7 | 11.0 |
| H.use | 6.9 | 7.0 | 5.1 | 6.4 | 5.6 | 6.3 | 5.1 | 6.2 |
| H.cost | 1246 | 1212 | 713 | 1034 | 860 | 1044 | 1234 | 2061 |
| H.aver | 170.5 | 166.3 | 140.0 | 161.7 | 152.4 | 166.3 | 104.3 | 119.8 |

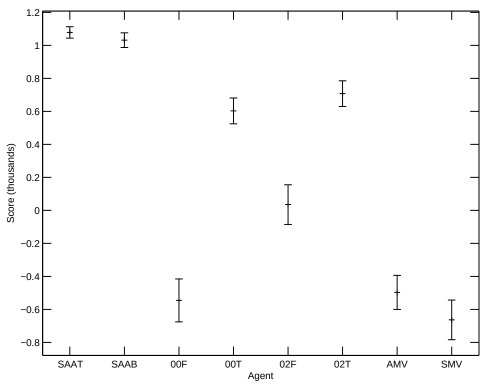
Table 22: Experiment with noise, $\sigma = 80$, $\lambda = -40$



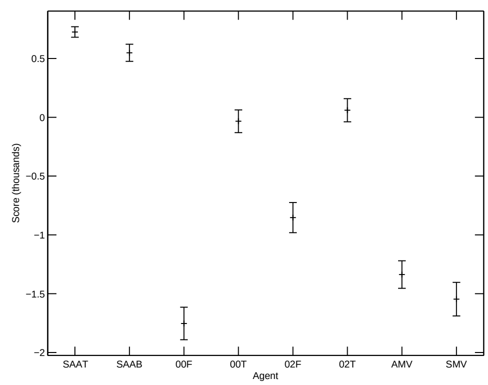

(a) $\lambda = -10$



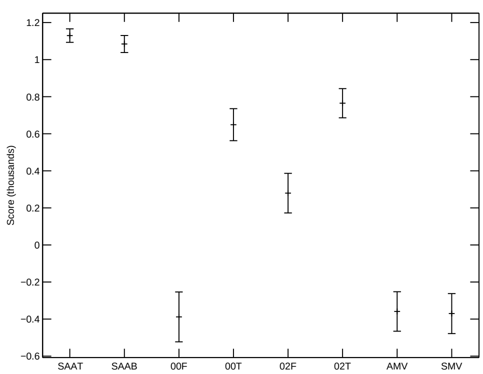

(b) $\lambda = 10$



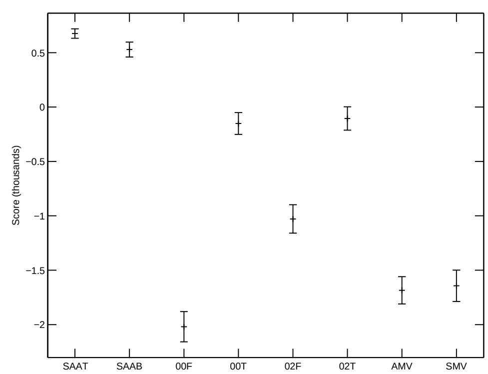

(c) $\lambda = -20$




(d) $\lambda = 20$

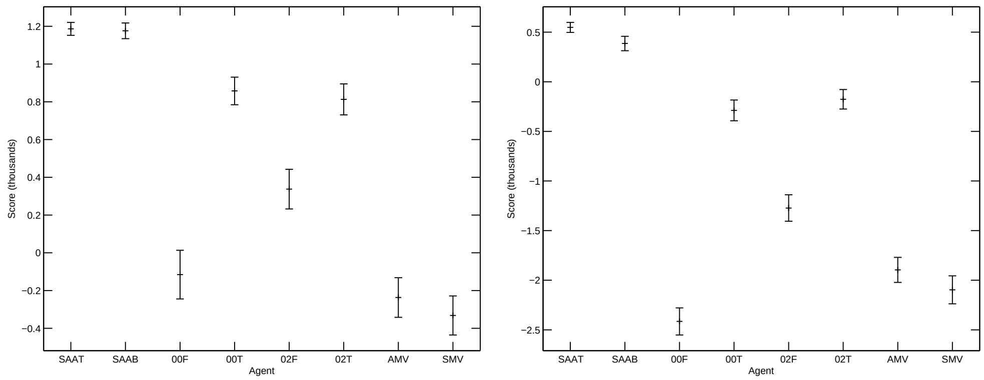



(e) $\lambda = -30$




(f) $\lambda = 30$

Figure 12: Confidence Intervals of score mean, experiment with noise, $\sigma = 80$, I




(a) $\lambda = -40$ (b) $\lambda = 40$

Figure 13: Confidence Intervals of score mean, experiment with noise, $\sigma = 80$, II

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 1129 | 1084 | -388 | 648 | 279 | 764 | -359 | -370 |
| Utility | 6942 | 6904 | 4816 | 6338 | 5658 | 6236 | 4840 | 6176 |
| Cost | 5813 | 5820 | 5204 | 5689 | 5378 | 5471 | 5199 | 6546 |
| Penalty | 512 | 514 | 308 | 397 | 345 | 373 | 168 | 427 |
| Null | 1.04 | 1.06 | 3.22 | 1.69 | 2.33 | 1.77 | 3.33 | 1.88 |
| F.unuse | 0.05 | 0.18 | 4.28 | 1.55 | 2.46 | 1.08 | 3.09 | 1.78 |
| F.use | 13.9 | 13.9 | 9.6 | 12.6 | 11.3 | 12.5 | 9.3 | 12.2 |
| F.cost | 4538 | 4572 | 4501 | 4605 | 4482 | 4399 | 4039 | 4558 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 497 | 477 | 343 | 429 | 338 | 384 | 340 | 482 |
| H.earn | 7.3 | 7.2 | 4.8 | 6.3 | 5.7 | 6.3 | 10.7 | 16.0 |
| H.unuse | 0.3 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 5.8 | 9.9 |
| H.use | 7.0 | 6.9 | 4.8 | 6.3 | 5.7 | 6.2 | 5.0 | 6.1 |
| H.cost | 1274 | 1247 | 704 | 1085 | 896 | 1072 | 1160 | 1988 |
| H.aver | 174.9 | 174.1 | 147.3 | 172.0 | 158.2 | 171.5 | 108.3 | 124.0 |

Table 23: Experiment with noise, $\sigma = 80$, $\lambda = -30$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 1078 | 1031 | -545 | 602 | 35 | 707 | -497 | -663 |
| Utility | 6954 | 6833 | 4649 | 6232 | 5376 | 6256 | 4616 | 5704 |
| Cost | 5876 | 5801 | 5195 | 5629 | 5341 | 5549 | 5113 | 6368 |
| Penalty | 510 | 505 | 287 | 363 | 307 | 374 | 159 | 386 |
| Null | 1.04 | 1.13 | 3.39 | 1.83 | 2.64 | 1.74 | 3.55 | 2.34 |
| F.unuse | 0.04 | 0.20 | 4.63 | 1.58 | 3.02 | 1.08 | 3.41 | 2.48 |
| F.use | 13.9 | 13.7 | 9.2 | 12.3 | 10.7 | 12.5 | 8.9 | 11.3 |
| F.cost | 4540 | 4531 | 4502 | 4527 | 4467 | 4417 | 3999 | 4482 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 501 | 468 | 328 | 424 | 322 | 375 | 328 | 435 |
| H.earn | 7.3 | 7.0 | 4.6 | 6.2 | 5.4 | 6.3 | 10.0 | 14.9 |
| H.unuse | 0.3 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 5.3 | 9.2 |
| H.use | 7.0 | 6.9 | 4.6 | 6.2 | 5.4 | 6.3 | 4.7 | 5.7 |
| H.cost | 1336 | 1271 | 693 | 1103 | 875 | 1131 | 1114 | 1885 |
| H.aver | 184.0 | 180.5 | 150.5 | 178.7 | 163.2 | 179.3 | 111.9 | 126.7 |

Table 24: Experiment with noise, $\sigma = 80$, $\lambda = -20$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 1010 | 921 | -913 | 460 | -194 | 590 | -715 | -642 |
| Utility | 6916 | 6749 | 4320 | 6114 | 5134 | 6153 | 4308 | 5635 |
| Cost | 5905 | 5827 | 5233 | 5653 | 5328 | 5563 | 5023 | 6277 |
| Penalty | 498 | 487 | 257 | 352 | 288 | 361 | 142 | 364 |
| Null | 1.09 | 1.22 | 3.72 | 1.95 | 2.90 | 1.87 | 3.86 | 2.42 |
| F.unuse | 0.08 | 0.34 | 5.53 | 1.87 | 3.57 | 1.37 | 4.03 | 2.72 |
| F.use | 13.8 | 13.6 | 8.6 | 12.1 | 10.2 | 12.3 | 8.3 | 11.2 |
| F.cost | 4520 | 4519 | 4580 | 4538 | 4477 | 4427 | 4003 | 4508 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 503 | 455 | 296 | 419 | 321 | 389 | 306 | 422 |
| H.earn | 7.1 | 6.9 | 4.3 | 6.0 | 5.1 | 6.1 | 8.8 | 13.4 |
| H.unuse | 0.2 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 4.4 | 7.8 |
| H.use | 6.9 | 6.8 | 4.3 | 6.0 | 5.1 | 6.1 | 4.3 | 5.6 |
| H.cost | 1385 | 1309 | 653 | 1115 | 851 | 1137 | 1021 | 1769 |
| H.aver | 193.9 | 189.5 | 152.7 | 184.4 | 166.8 | 185.0 | 116.4 | 131.8 |

Table 25: Experiment with noise, $\sigma = 80$, $\lambda = -10$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 744 | 634 | -1744 | 21 | -805 | 151 | -1374 | -1335 |
| Utility | 6787 | 6599 | 3329 | 5753 | 4478 | 5734 | 3369 | 4520 |
| Cost | 6043 | 5964 | 5074 | 5731 | 5284 | 5583 | 4744 | 5855 |
| Penalty | 491 | 480 | 184 | 303 | 223 | 288 | 105 | 290 |
| Null | 1.21 | 1.36 | 4.71 | 2.34 | 3.57 | 2.34 | 4.75 | 3.50 |
| F.unuse | 0.18 | 0.63 | 7.36 | 2.68 | 4.80 | 2.11 | 5.81 | 4.80 |
| F.use | 13.6 | 13.3 | 6.6 | 11.3 | 8.9 | 11.3 | 6.5 | 9.0 |
| F.cost | 4471 | 4523 | 4528 | 4550 | 4440 | 4367 | 4003 | 4485 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 493 | 437 | 229 | 395 | 272 | 360 | 223 | 313 |
| H.earn | 6.9 | 6.7 | 3.3 | 5.7 | 4.4 | 5.7 | 5.7 | 9.5 |
| H.unuse | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.4 | 5.0 |
| H.use | 6.8 | 6.6 | 3.3 | 5.7 | 4.4 | 5.7 | 3.3 | 4.5 |
| H.cost | 1572 | 1441 | 546 | 1181 | 844 | 1216 | 741 | 1370 |
| H.aver | 226.9 | 215.6 | 166.3 | 208.7 | 190.6 | 213.7 | 130.1 | 143.9 |

Table 26: Experiment with noise, $\sigma = 80$, $\lambda = 10$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 725 | 548 | -1753 | -33 | -853 | 60 | -1337 | -1546 |
| Utility | 6749 | 6579 | 3255 | 5697 | 4433 | 5579 | 3320 | 4249 |
| Cost | 6024 | 6031 | 5008 | 5731 | 5287 | 5519 | 4657 | 5796 |
| Penalty | 505 | 488 | 176 | 290 | 221 | 279 | 97 | 271 |
| Null | 1.21 | 1.39 | 4.78 | 2.40 | 3.61 | 2.48 | 4.80 | 3.77 |
| F.unuse | 0.22 | 0.83 | 7.39 | 2.76 | 4.95 | 2.39 | 5.72 | 5.39 |
| F.use | 13.6 | 13.2 | 6.4 | 11.2 | 8.8 | 11.0 | 6.4 | 8.5 |
| F.cost | 4481 | 4567 | 4493 | 4538 | 4466 | 4365 | 3941 | 4502 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 469 | 458 | 214 | 388 | 261 | 338 | 214 | 290 |
| H.earn | 6.9 | 6.7 | 3.2 | 5.6 | 4.4 | 5.5 | 5.8 | 9.4 |
| H.unuse | 0.1 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 2.5 | 5.1 |
| H.use | 6.8 | 6.6 | 3.2 | 5.6 | 4.4 | 5.5 | 3.3 | 4.2 |
| H.cost | 1542 | 1464 | 515 | 1192 | 821 | 1153 | 716 | 1294 |
| H.aver | 223.0 | 218.5 | 160.3 | 213.0 | 187.0 | 208.4 | 124.5 | 138.4 |

Table 27: Experiment with noise, $\sigma = 80$, $\lambda = 20$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 675 | 528 | -2019 | -150 | -1028 | -105 | -1684 | -1643 |
| Utility | 6825 | 6624 | 2993 | 5576 | 4223 | 5544 | 2975 | 4027 |
| Cost | 6149 | 6096 | 5013 | 5726 | 5251 | 5649 | 4660 | 5671 |
| Penalty | 505 | 474 | 166 | 276 | 203 | 286 | 87 | 245 |
| Null | 1.14 | 1.35 | 5.04 | 2.53 | 3.83 | 2.52 | 5.13 | 4.00 |
| F.unuse | 0.22 | 0.78 | 8.01 | 2.99 | 5.29 | 2.69 | 6.67 | 5.86 |
| F.use | 13.7 | 13.3 | 5.9 | 10.9 | 8.3 | 11.0 | 5.7 | 8.0 |
| F.cost | 4532 | 4576 | 4527 | 4528 | 4428 | 4437 | 4033 | 4505 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 469 | 450 | 201 | 381 | 261 | 348 | 191 | 273 |
| H.earn | 7.0 | 6.7 | 3.0 | 5.5 | 4.2 | 5.5 | 4.8 | 8.1 |
| H.unuse | 0.1 | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 1.9 | 4.1 |
| H.use | 6.9 | 6.7 | 3.0 | 5.5 | 4.2 | 5.5 | 2.9 | 4.0 |
| H.cost | 1617 | 1520 | 486 | 1198 | 824 | 1212 | 627 | 1166 |
| H.aver | 232.3 | 226.3 | 164.5 | 219.1 | 197.8 | 220.2 | 131.5 | 144.2 |

Table 28: Experiment with noise, $\sigma = 80$, $\lambda = 30$

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 548 | 386 | -2415 | -288 | -1272 | -176 | -1895 | -2096 |
| Utility | 6729 | 6402 | 2524 | 5417 | 3908 | 5455 | 2679 | 3401 |
| Cost | 6181 | 6015 | 4939 | 5705 | 5180 | 5631 | 4575 | 5498 |
| Penalty | 494 | 447 | 130 | 253 | 180 | 249 | 75 | 211 |
| Null | 1.25 | 1.60 | 5.52 | 2.70 | 4.14 | 2.62 | 5.42 | 4.63 |
| F.unuse | 0.38 | 1.03 | 8.92 | 3.22 | 5.82 | 2.77 | 7.21 | 7.06 |
| F.use | 13.5 | 12.8 | 5.0 | 10.6 | 7.7 | 10.8 | 5.2 | 6.7 |
| F.cost | 4508 | 4498 | 4510 | 4490 | 4401 | 4395 | 4020 | 4488 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 477 | 445 | 178 | 372 | 229 | 327 | 177 | 239 |
| H.earn | 6.8 | 6.4 | 2.5 | 5.3 | 3.9 | 5.4 | 3.9 | 6.6 |
| H.unuse | 0.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.3 | 3.3 |
| H.use | 6.7 | 6.4 | 2.5 | 5.3 | 3.9 | 5.4 | 2.6 | 3.4 |
| H.cost | 1673 | 1517 | 429 | 1215 | 780 | 1236 | 555 | 1010 |
| H.aver | 244.4 | 235.8 | 173.7 | 229.4 | 202.1 | 228.5 | 142.0 | 152.3 |

Table 29: Experiment with noise, $\sigma = 80$, $\lambda = 40$

## C. Experiment with Equilibrium Prediction

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 944 | 863 | -338 | 562 | 707 | 820 | -1353 | -1290 |
| Utility | 6536 | 6391 | 4756 | 6002 | 4617 | 5104 | 4528 | 4818 |
| Cost | 5592 | 5528 | 5094 | 5440 | 3909 | 4284 | 5881 | 6108 |
| Penalty | 426 | 423 | 261 | 328 | 133 | 178 | 198 | 261 |
| Null | 1.45 | 1.59 | 3.28 | 2.06 | 3.56 | 3.06 | 3.64 | 3.32 |
| F.unuse | 0.00 | 0.22 | 3.61 | 1.21 | 0.32 | 0.18 | 3.78 | 3.73 |
| F.use | 13.1 | 12.8 | 9.4 | 11.9 | 8.9 | 9.9 | 8.7 | 9.4 |
| F.cost | 4255 | 4235 | 4239 | 4253 | 2991 | 3272 | 4062 | 4256 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 417 | 408 | 301 | 391 | 310 | 339 | 367 | 396 |
| H.earn | 6.6 | 6.4 | 4.7 | 5.9 | 4.4 | 4.9 | 11.2 | 11.1 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 6.7 | 6.4 |
| H.use | 6.5 | 6.4 | 4.7 | 5.9 | 4.4 | 4.9 | 4.5 | 4.7 |
| H.cost | 1336 | 1294 | 856 | 1186 | 918 | 1012 | 1820 | 1852 |
| H.aver | 203.6 | 201.8 | 181.5 | 199.8 | 207.0 | 204.6 | 162.6 | 167.2 |

Table 30: Experiment with equilibrium, decision-theoretic setting, B(32, 0.5) players

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 2516 | 2019 | 904 | 2520 | 1989 | 2492 | 1447 | 1233 |
| Utility | 8579 | 8010 | 6717 | 8577 | 7703 | 8387 | 7276 | 7144 |
| Cost | 6063 | 5991 | 5813 | 6056 | 5713 | 5895 | 5829 | 5911 |
| Penalty | 46 | 56 | 53 | 61 | 363 | 230 | 82 | 57 |
| Null | 0.00 | 0.51 | 1.72 | 0.00 | 0.54 | 0.00 | 1.13 | 1.30 |
| F.unuse | 0.00 | 1.02 | 3.43 | 0.00 | 1.08 | 0.00 | 2.16 | 2.60 |
| F.use | 16.0 | 15.0 | 12.6 | 16.0 | 14.9 | 16.0 | 13.7 | 13.4 |
| F.cost | 5200 | 5200 | 5200 | 5200 | 5200 | 5200 | 5165 | 5201 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 627 | 576 | 488 | 639 | 607 | 618 | 490 | 499 |
| H.earn | 16.0 | 15.4 | 12.8 | 15.1 | 11.2 | 13.8 | 17.2 | 15.5 |
| H.unuse | 0.5 | 1.2 | 1.8 | 0.0 | 0.3 | 0.0 | 5.1 | 3.6 |
| H.use | 15.5 | 14.2 | 11.0 | 15.1 | 10.9 | 13.8 | 12.1 | 12.0 |
| H.cost | 863 | 791 | 613 | 856 | 513 | 695 | 664 | 710 |
| H.aver | 53.8 | 51.4 | 47.8 | 56.6 | 45.7 | 50.6 | 38.7 | 45.7 |

Table 31: Experiment with equilibrium, game-theoretic setting, 8 players

| Agent | SAAT | SAAB | TMU | TMU* | BE | BE* | AMU | SMU |
|---|---|---|---|---|---|---|---|---|
| Score | 1041 | 1035 | 418 | 867 | 899 | 943 | -721 | -670 |
| Utility | 6520 | 6519 | 5653 | 6304 | 4765 | 5089 | 5664 | 6012 |
| Cost | 5479 | 5484 | 5235 | 5437 | 3865 | 4145 | 6385 | 6683 |
| Penalty | 437 | 427 | 316 | 382 | 137 | 181 | 267 | 341 |
| Null | 1.46 | 1.47 | 2.39 | 1.71 | 3.41 | 3.07 | 2.52 | 2.12 |
| F.unuse | 0.00 | 0.05 | 1.83 | 0.58 | 0.07 | 0.02 | 1.56 | 1.34 |
| F.use | 13.1 | 13.1 | 11.2 | 12.6 | 9.2 | 9.9 | 11.0 | 11.8 |
| F.cost | 4250 | 4261 | 4244 | 4275 | 3002 | 3215 | 4070 | 4257 |
| F.aver | 325 | 325 | 325 | 325 | 325 | 325 | 325 | 325 |
| H.bonus | 420 | 420 | 357 | 402 | 317 | 336 | 447 | 477 |
| H.earn | 6.6 | 6.6 | 5.6 | 6.3 | 4.6 | 4.9 | 14.0 | 14.3 |
| H.unuse | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 8.4 | 8.4 |
| H.use | 6.5 | 6.5 | 5.6 | 6.3 | 4.6 | 4.9 | 5.7 | 5.9 |
| H.cost | 1229 | 1223 | 991 | 1162 | 863 | 930 | 2315 | 2426 |
| H.aver | 186.7 | 186.5 | 176.6 | 184.9 | 188.3 | 188.4 | 164.9 | 169.8 |

Table 32: Experiment with equilibrium, game-theoretic setting, B(32, 0.5) players