

# Comparison of Bidding Algorithms for Simultaneous Auctions

Seong Jae Lee

Introduction

# Bidding Problem

- Simultaneous Auctions
- Substitutable & Complementary Goods



Miami Beach

# Bidding Problem: Goal

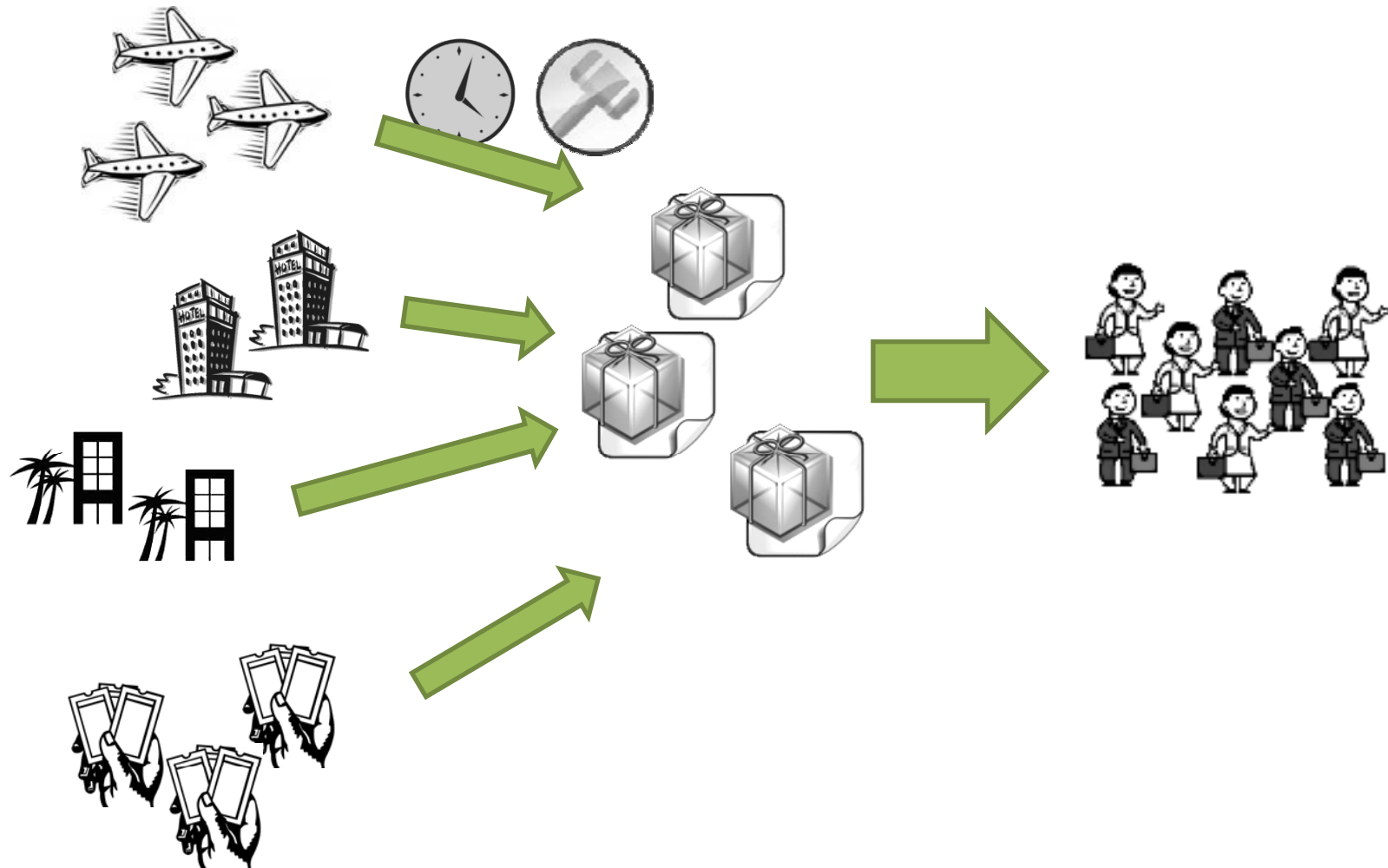
- The goal of bidding problem is to find a set of bids  $B$  that maximizes:

$$\int_s p(s)v(s, B)ds$$

- $s$  : clearing price.
- $p(s)$  : probability that the clearing price is  $s$ .
- $v(s, B)$  : value when the clearing price is  $s$ , and bid is  $B$ .

Introduction

# Trading Agent Competition



# Algorithms

- Sample Average Approximation
- Marginal Value Bidding

Name	Performance	Algorithm
ATTac01	2000 1 <sup>st</sup> , 2003 1 <sup>st</sup>	Marginal Value
Walverine	2004 2 <sup>nd</sup> , 2005 3 <sup>rd</sup> , 2006 2 <sup>nd</sup>	Marginal Value
RoxyBot	2000 2 <sup>nd</sup> , 2002 final	Marginal Value
RoxyBot	2005 final, 2006 1 <sup>st</sup>	SAA

# Review: the Goal

- The goal of bidding problem is to find a set of bids  $B$  that maximizes:

$$\int_s p(s)v(s, B)ds$$

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# Sample Average Approximation

- SAA algorithm samples  $S$  scenarios from clearing price distribution model.

- Find a set of bids  $B$  that maximizes:

$$\frac{1}{|S|} \sum_{s \in S} v(s, B)$$

- $S$  : a set of sampled clearing prices.

# Sample Average Approximation

- There are infinitely many solutions!
  - e.g.  $S=1$ ,  $s=100$ ,  
if  $B > s$ ,  $v(s, B) = 1000 - s$ , else  $v(s, B) = 0$ .
  - $B$  can be any number between 100 and 1000.

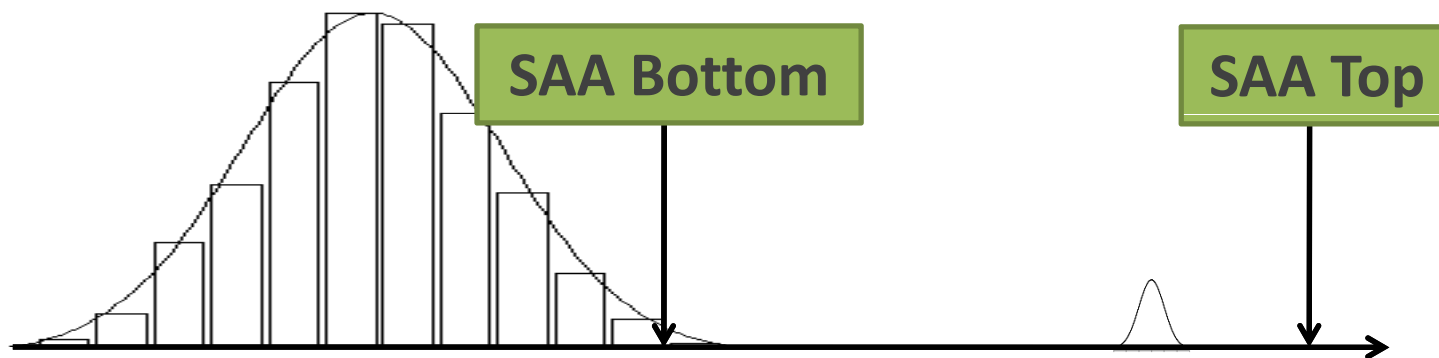
- SAA Bottom: maximize  $\frac{1}{|S|} \sum_{s \in S} v(s, B) - \epsilon B$
- SAA Top: maximize

$$\frac{1}{|S|} \sum_{s \in S} v(s, B) + \epsilon B, \quad b < c \quad \forall b \in B$$



# Sample Average Approximation

- Defect
  - The highest bid SAA Bottom considers submitting may be below clearing price.
  - SAA Top may pay more than the highest price it expects.



# Marginal Value based Algorithms

- Marginal Value of a good: the additional value derived from owning the good relative to the set of goods you can buy.
- Characterization Theorem [Greenwald]
  - $MV(g) > s$  if  $g$  is in all optimal sets.
  - $MV(g) = s$  if  $g$  is in some optimal sets.
  - $MV(g) < s$  if  $g$  is not in any optimal sets.

# Marginal Value based Algorithms

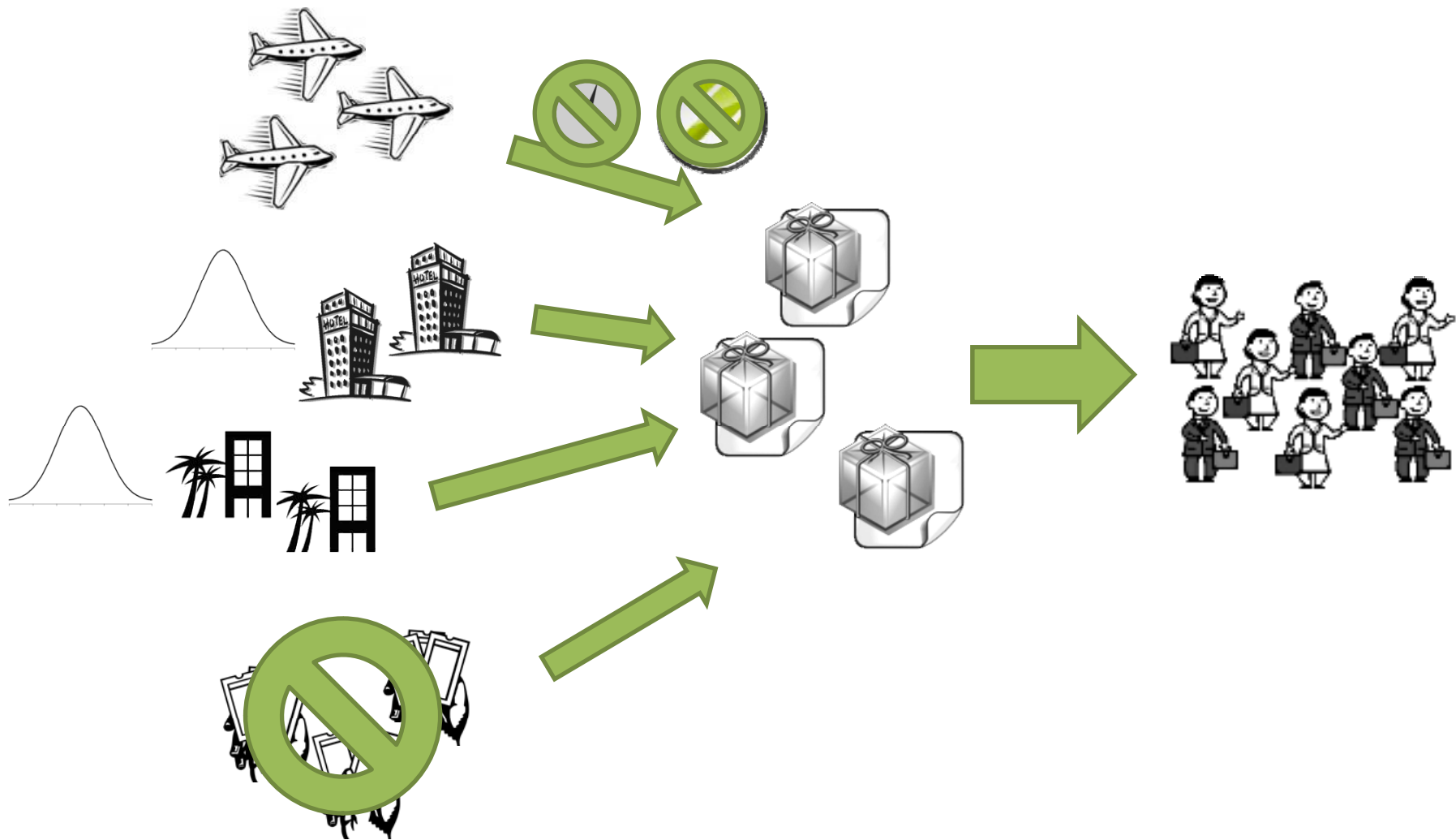
- Use MV based algorithms which performed well in the TAC:
  - TMU/TMU\*: RoxyBot 2000
  - BE/BE\* : RoxyBot 2002
  - AMU/SMU : ATTAC

# Experiments

- Decision-Theoretic Setting
  - Prediction = Clearing Price (normal dist.)
  - Prediction  $\sim$  Clearing Price (normal dist.)
- Game-Theoretic Setting
  - Prediction  $\sim$  Clearing Price (CE price)

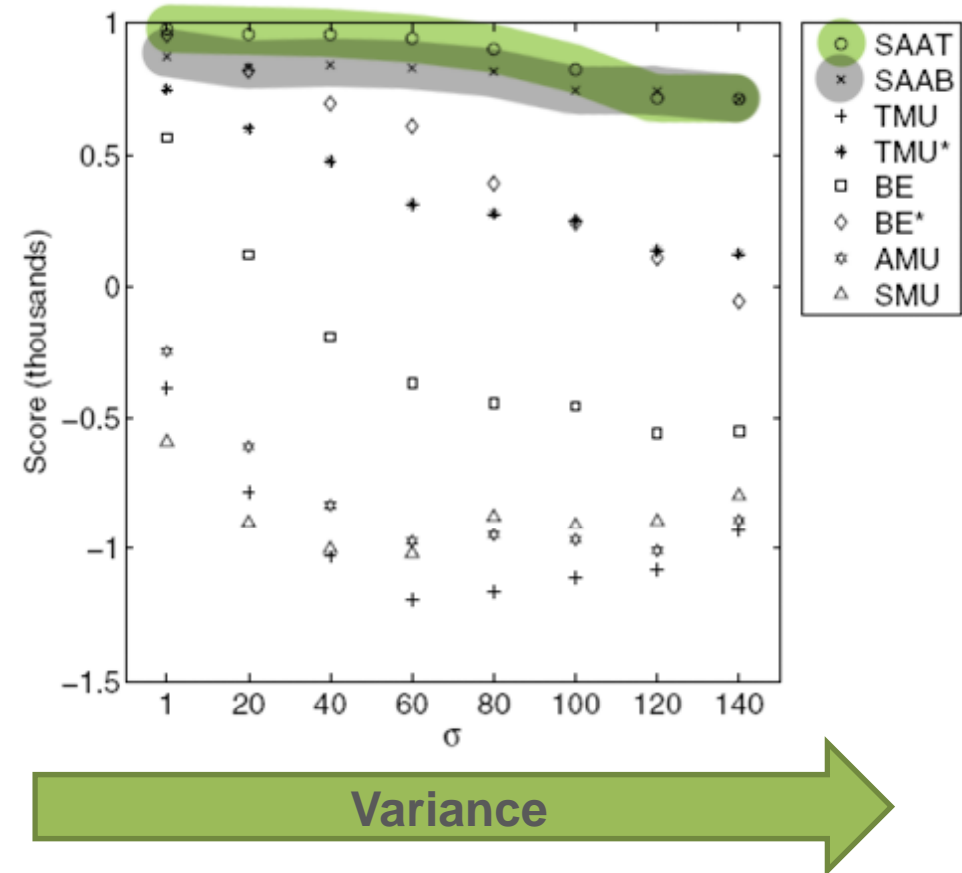
Experiments

# 1. Decision-Theoretic (perfect)

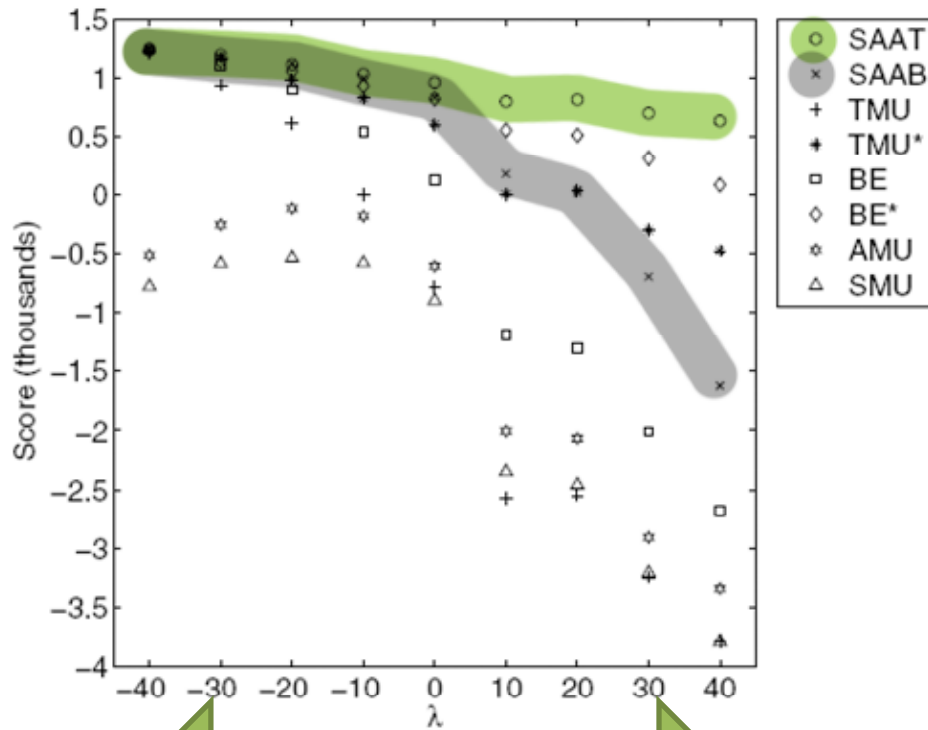


# 1. Decision-Theoretic (perfect)

- SAAs are more tolerant to variance
- SAAT ~ SAAB at a high variance



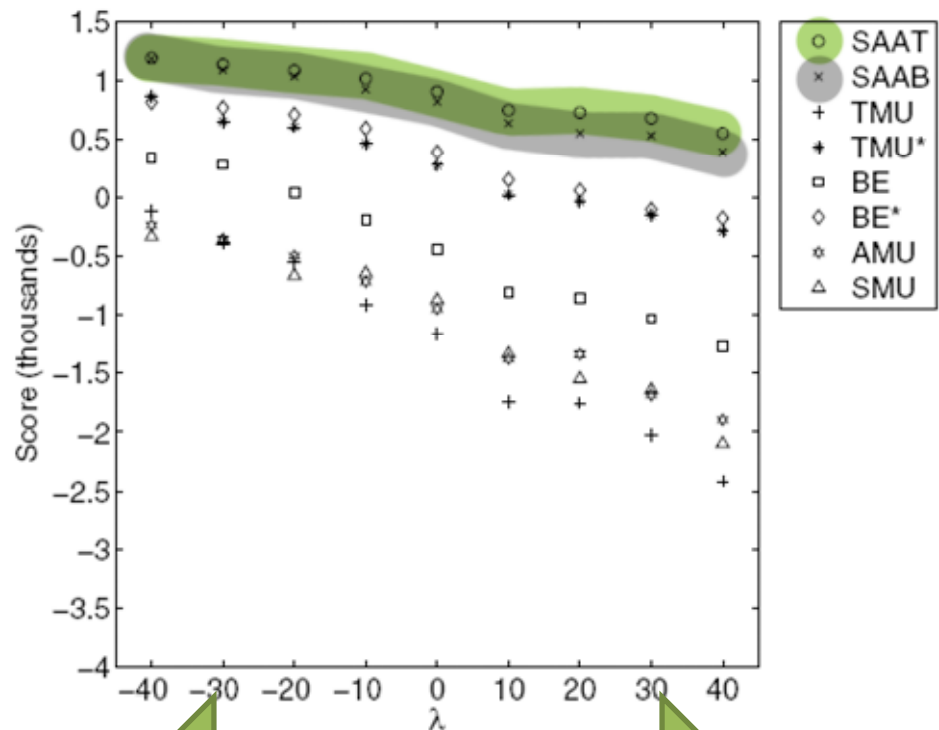
# 2. Decision-Theoretic (noise)



Over-prediction

Under-prediction

Low Variance



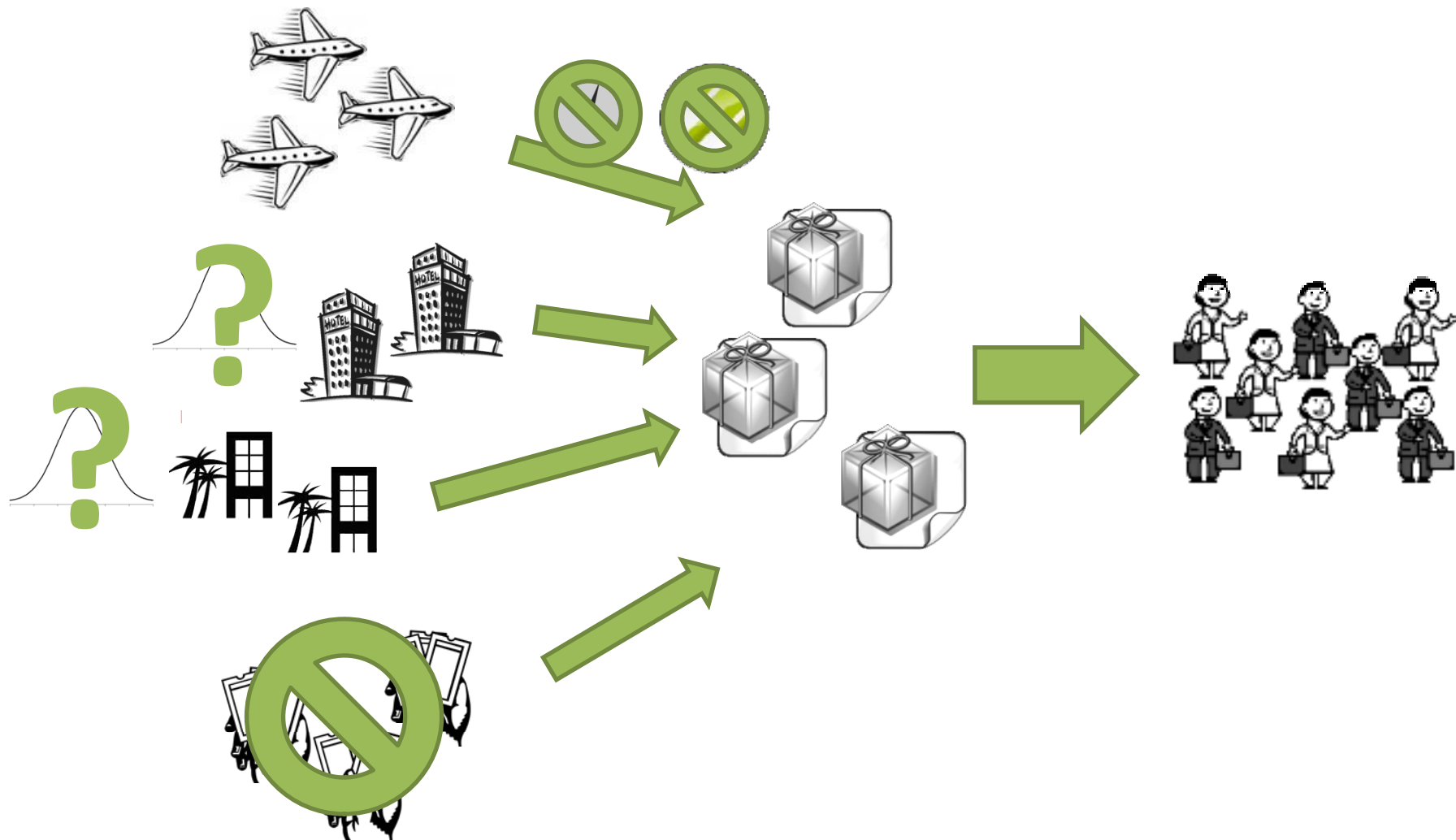
Over-prediction

Under-prediction

High Variance

Experiments

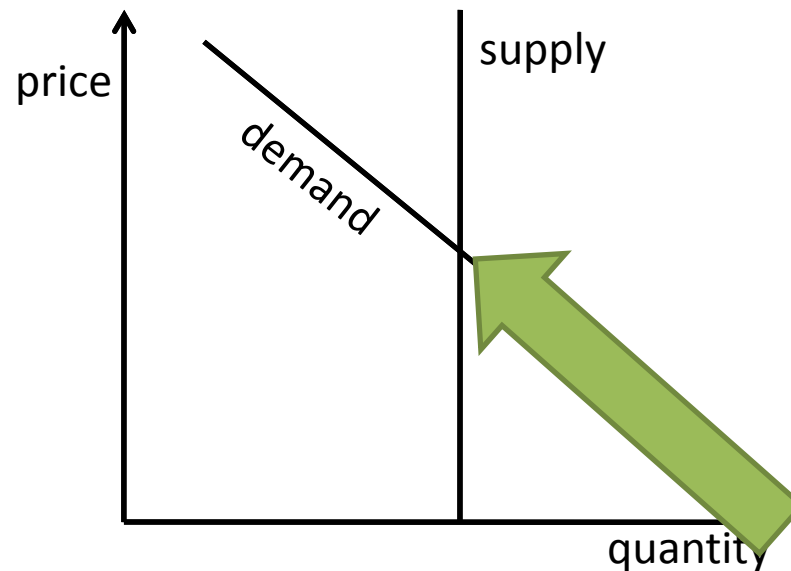
# 3. Game-Theoretic (CE prices)



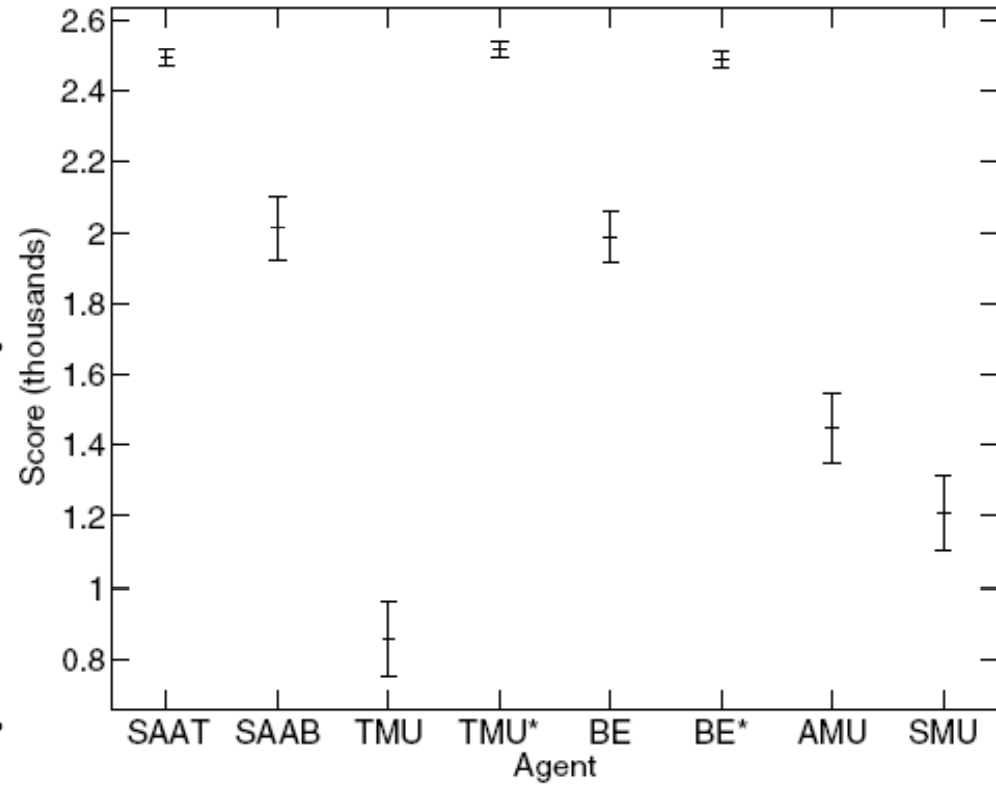
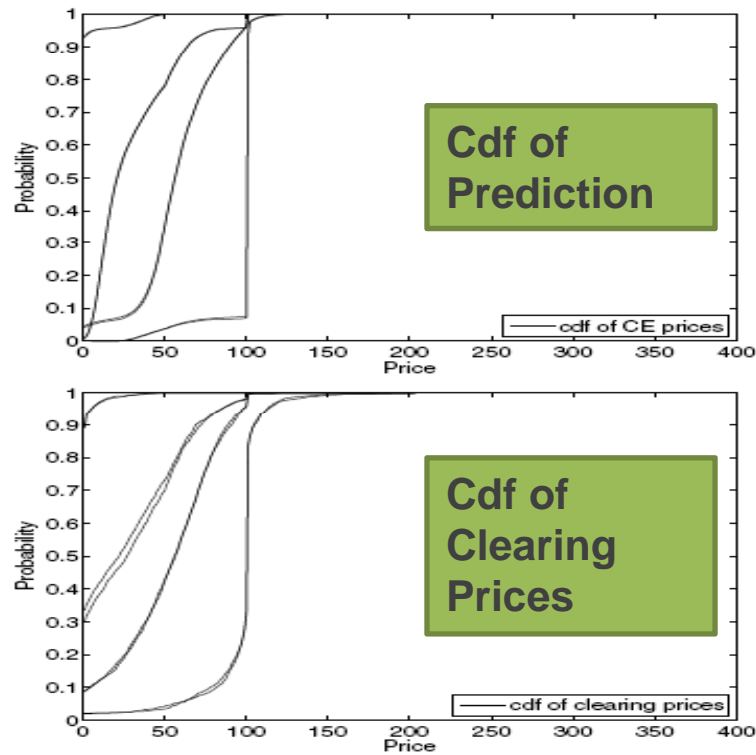


# 3. Game-Theoretic (CE prices)

- Competitive Equilibrium [Wellman '04]
- $P_{n+1} = P_n + \text{MAX}(0, \alpha P_n (\text{demand} - \text{supply}))$

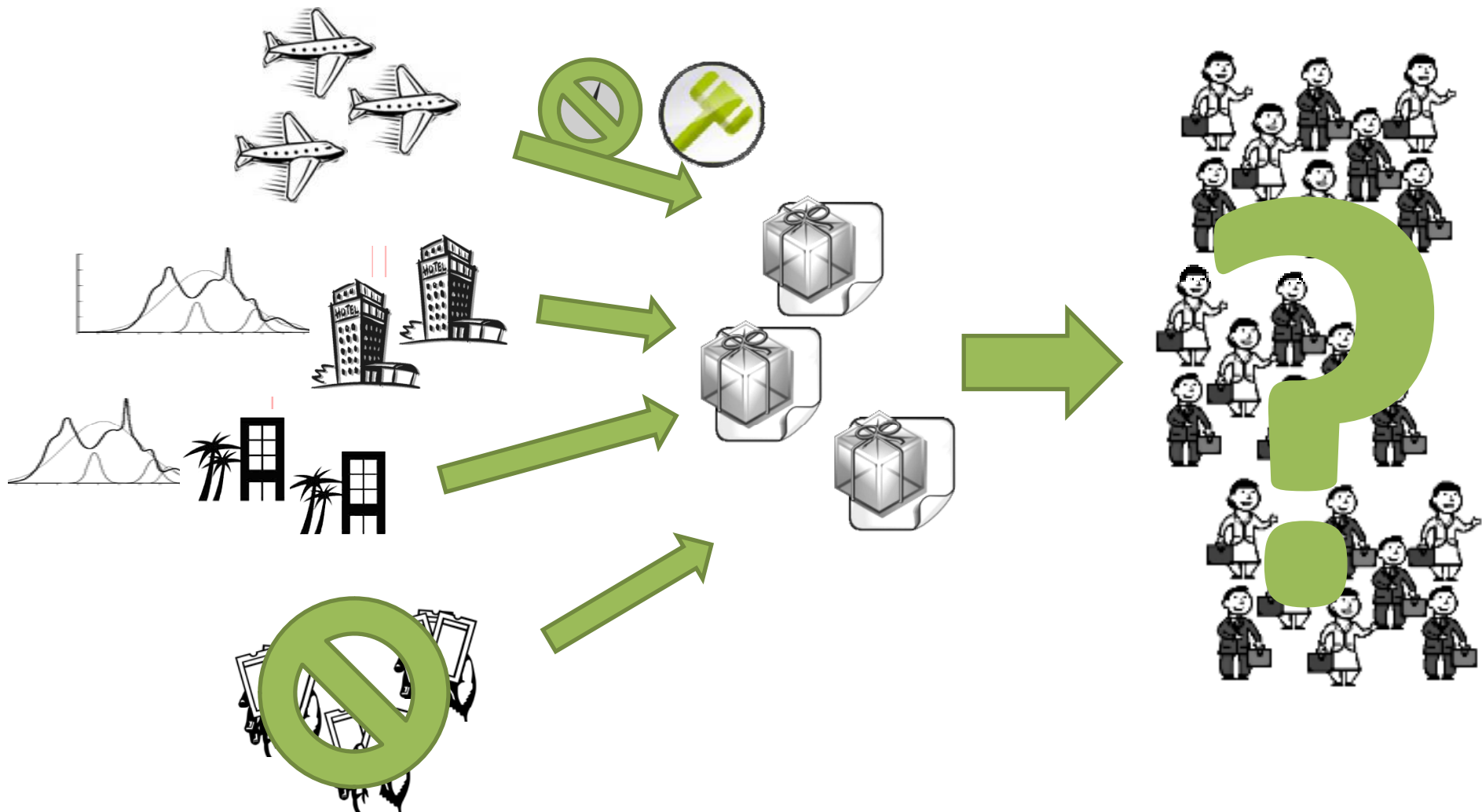


# 3. Game-Theoretic (CE prices)



Experiments

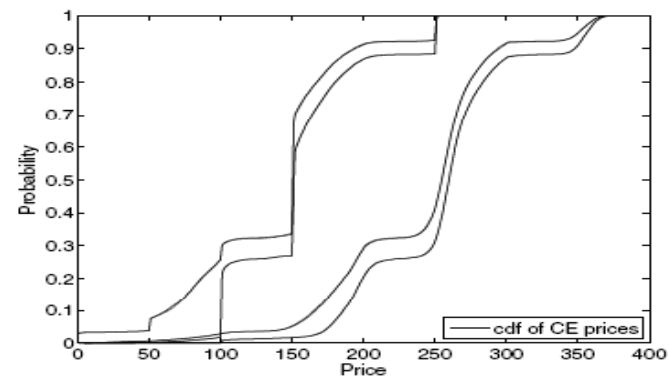
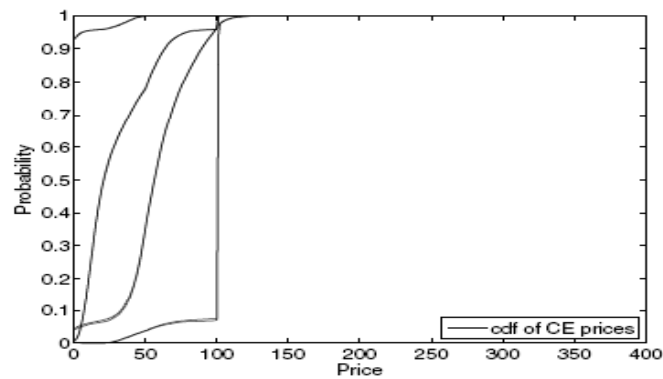
# 3. Game-Theoretic (CE prices)



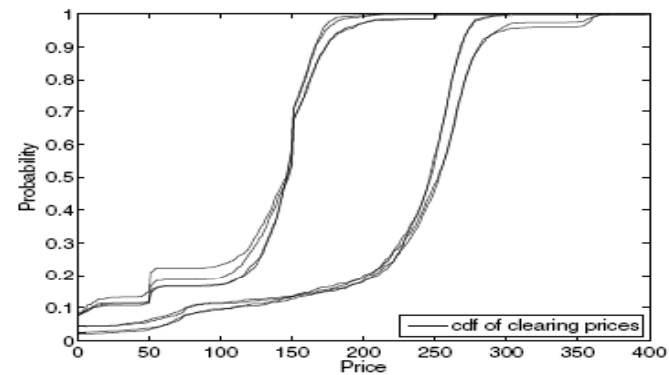
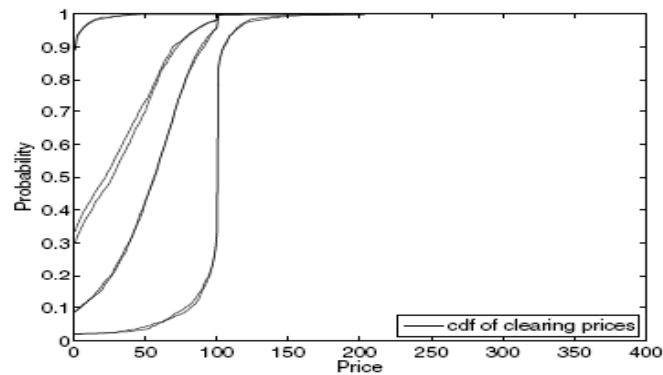
Experiments

# 3. Game-Theoretic (CE prices)

Cdf of Prediction



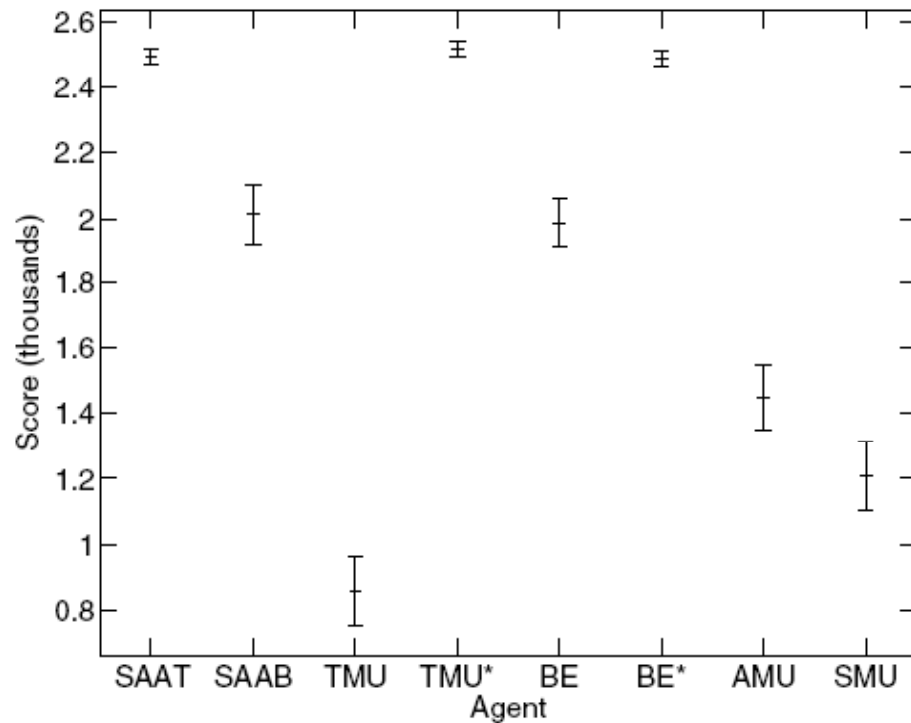
Cdf of Clearing Prices



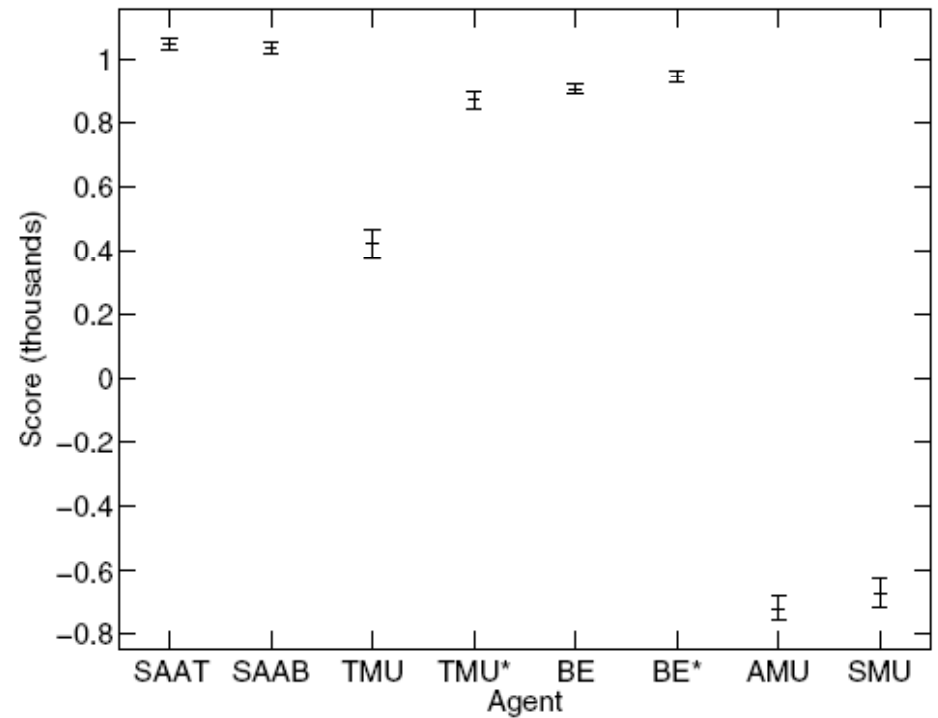
Low Variance

High Variance

# 3. Game-Theoretic (CE prices)



Low Variance



High Variance

# Conclusion

- Sample Average Approximation
  - Optimal for decision-theoretic setting, with infinite number of scenarios.
  - More tolerant to variance.
  - More tolerant to noise.
    - SAA Top is tolerant to noise in general.
    - SAA Bottom is tolerant to noise in high variance.
  - Showed a better performance even in a game-theoretic setting.

Questions?

# Acknowledgements

- Andries van Dam
- Amy Greenwald
- Victor Naroditskiy
- Meinolf Sellmann